Math 104, Lecture 2 Problem Set 10 Spring 2011

Due the week of March 28, 2011, in recitation.

Reminders:

1. Exam #3 will be held in class on Friday, April 1. It will cover the material from section 9.3 to 9.5, and from 12.1 to 12.6.

2. Extra credit will be given to those who hand in Sample Exam 3 to their TA by 1pm on Thursday, March 31. There is also an additional set of study problems that can be used to help prepare for the exam. Both can be found on the exam web page for this lecture in advance of the exam.

Read Stewart, sections 12.3-12.5.

For each of these sections, work through the first five core problems, but do not hand them in.

The problems below are to be handed in.

Part A: Do the following problems from Stewart:

Section 12.1: 58, 67, 72.
Section 12.2: 38, 43, 49, 60, 73.
Section 12.3: 3, 7, 12, 13, 33.
Section 12.4: 3, 5, 28, 29, 37.
Section 12.5: 6, 9, 13, 23.

Part B: Do the following problems:

1. Let $a_1, a_2, a_3, \ldots$ be real numbers. Determine whether each of the following statements about *sequences* is (always) true, and in each case explain why. Also, for each false statement, given an example that shows that it does not always hold.

a) If $\{a_n\}$ converges, then so does $\{a_n/2\}$.

b) If $\{a_n\}$ converges, then so does $\{a_n + 1\}$.

c) If $\{a_n\}$ converges, then so does $\{a_{2n}\}$.

d) If $\{a_{2n}\}$ converges, then so does $\{a_n\}$.

e) If each $a_n \neq 0$, and if $\{a_n\}$ converges, then so does $\{1/a_n\}$.

f) If $\{|a_n|\}$ converges, then so does $\{a_n\}$.

g) If $\{a_n\}$ is an increasing sequence, then $\{a_n\}$ diverges.

2. Let $a_1, a_2, a_3, \ldots$ be real numbers. Determine whether each of the following statements about *series* is (always) true, and in each case explain why. Also, for each false statement, given an example that shows that it does not always hold.

a) If $\sum_{n=1}^{\infty} a_n$ converges, then so does $\sum_{n=1}^{\infty} a_n/2$.

b) If $\sum_{n=1}^{\infty} a_n$ converges, then so does $\sum_{n=1}^{\infty}(a_n + 1)$.

c) If $\sum_{n=1}^{\infty} a_n$ converges, then so does $\sum_{n=1}^{\infty} a_{2n}$.

d) If each $a_n > 0$ and if $\sum_{n=1}^{\infty} a_n$ converges, then so does $\sum_{n=1}^{\infty} a_{2n}$.

e) If $\sum_{n=1}^{\infty} a_{2n}$ converges, then so does $\sum_{n=1}^{\infty} a_n$.

f) If each $a_n > 0$ and if $\sum_{n=1}^{\infty} a_n$ converges, then so does $\sum_{n=1}^{\infty} a_n^2$.

g) If $\sum_{n=1}^{\infty} a_n$ converges and each $a_n \neq 0$, then $\sum_{n=1}^{\infty} 1/a_n$ diverges.

3. Let $a_1, a_2, a_3, \ldots$ be real numbers. Determine whether each of the following statements about *sequences and series* is (always) true, and in each case explain why. Also, for each false statement, given an example that shows that it does not always hold.

a) If $\{a_n\}$ converges, then so does $\sum_{n=1}^{\infty} a_n$.

b) If $\{a_n\}$ converges to 0, then so does $\sum_{n=1}^{\infty} a_n$.

c) If $\sum_{n=1}^{\infty} a_n$ converges, then so does $\{a_n\}$.

d) If $\sum_{n=1}^{\infty} a_n$ converges to 1, then so does $\{a_n\}$.

e) If each $a_n > 1$ and if $\{a_n\}$ converges, then $\sum_{n=1}^{\infty} a_n$ diverges.

f) If $\{a_n\}$ is an increasing sequence of positive numbers, then $\sum_{n=1}^{\infty} a_n$ diverges.

g) If each a_n is an integer, and if $\sum_{n=1}^{\infty} a_n$ converges, then the sequence $\{a_n\}$ is eventually constant.