Math 104, Lecture 2 Problem Set 9 Spring 2011

Due the week of March 21, 2011, in recitation.

Read Stewart, sections 12.1-12.2.

Work through the following core problems, but do not hand them in:
Section 9.5: core problems beginning with #8.
Section 12.1: first eight core problems.
Section 12.2: first eight core problems.

The problems below are to be handed in.

Part A: Do the following problems from Stewart:

Section 9.3: 22, 23, 25.
Section 9.5: 1, 3, 6, 9.
Section 12.1: 15, 21, 27, 54, 56, 62, 63.
Section 12.2: 13, 15, 16, 21, 24, 27, 36.

Part B: Do the following problems:

1. Given a statistical quantity that has a normal distribution, the *z-score* of a given observation is the number of standard deviations by which it exceeds the median. (E.g. $z = 2$ for a quantity that exceeds the median by two standard deviations, and $z = -1$ for a quantity that is less than the median by one standard deviation.)

a) What is $P(Z > 0)$, if Z represents the z-score of a given observation?

b) What is the $\lim_{z\to\infty} P(Z > z)$?

c) What is the $\lim_{z\to-\infty} P(Z > z)$?

d) Write down an integral corresponding to $P(Z > 1.5)$. Then estimate its value. (To do this you can either use approximate methods of integration, or use computer software such as Maple or Mathematica, or look up z-scores and percentiles on the web to find a table or converter.)

2. In 1941, the 200-800 scale for the SAT was established in the following manner. Each student's exam that year was assigned a raw score based on the number of questions answered correctly and incorrectly. These raw scores were then assigned scores on the 200-800 scale in such a way that they fell on a normal distribution with a median of 500 and a standard deviation of 100. Scores that exceeded the median by more than three standard deviations were rounded down to 800, and those that fell below the median by more than three standard deviations were rounded up to 200.

a) What proportion of these students had scores of at least 650? Express this as an integral and approximate its value. (Hint: Use what you did for problem 1.)

b) Do the same for the proportion of students whose scores were so high that they were rounded down to 800.

3. (Continuation of problem 2.) In years after 1941, the correspondence between the raw scores and the scores on the 200-800 scale was kept the same as in 1941, using the 1941 cohort as the basis, so that scores between one year and another could be compared. But the median score declined over time; and so the correspondence was changed in 1995, using the 1990 cohort as the new basis, and changing the standard deviation to 110.

a) Explain how this change affected the scores on the 200-800 scale. In particular, explain what happened to the score that is assigned to an exam whose raw score was equal to the 1990 median.

b) Redo problem 2 for the new scale and the new cohort. Did the proportions go up or down? Why?

4. For each of the following sequences $\{a_n\}$, determine its behavior (in particular, whether it is increasing, decreasing, constant, bounded above, bounded below, convergent, divergent). If it is convergent, find the limit.

a) $a_n = \sin(1/n)$ b) $a_n = \sin(n)$ c) $a_n = \sin(n)/n$

d) $a_n = n^2 e^{-n}$. e) $a_n = \ln(\ln(n+1))$. f) $a_n = e^{\cos(1/n)}$.

g) $a_n = 10^{-n}[10^n\sqrt{2}]$, where the notation $[x]$ denotes the greatest integer that is $\leq x$. (For example, $[1.6] = 1$, $[-1.6] = -2$, and $[2] = 2$.)

5. Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges, where a_n is given as below. If it does, find the limit.

a) $a_n = (-1/4)^n$ b) $a_n = .999^n$ c) $a_n = 1.001^n$

d) $\{a_n\}$ is the sequence $1, 1, -1, 1, 1, -1, 1, 1, -1, \ldots$.

e) $\{a_n\}$ is the sequence $1, 0, 1/2, 0, 1/4, 0, 1/8, 0, \ldots$.

6. a) For any real number $x \geq 0$, define a sequence $a_n(x)$ by $a_1(x) = x$ and $a_{n+1}(x) = a_n(x)^2$. For which values of x is the sequence increasing? decreasing? constant? convergent? When it converges, what is the limit? (First experiment with the values $x = 0, 1/2, 1, 2$, evaluating the first several terms to see what behavior you observe.)

b) Redo part (a) for the sequence $b_n(x)$ defined by $b_1(x) = x$ and $b_{n+1}(x) = \sqrt{b_n(x)}$.

c) Redo part (a) for the sequence $c_n(x)$ defined by $c_1(x) = x$ and $c_{n+1}(x) = \sqrt{c_n(x) + 1}$. (First experiment with the values $x = 0$ and 8.)