Math 104, Lecture 2 Problem Set 7 Spring 2011

Due the week of February 28, 2011, in recitation.

Reminders:

1. Exam #2 will be held in class on Monday, Feb. 28. It will cover the material from section 8.3 up through section 9.2, including section 8.5.

2. Extra credit will be given to those who hand in Sample Exam 2 to their TA by 1pm on Friday, Feb. 25. There is also an additional set of study problems that can be used to help prepare for the exam. Both are available on the exam web page for this lecture.

Read Stewart, sections 9.1 and 9.2.

For each of these two sections, work through the first four core problems, but do not hand them in.

The problems below are to be handed in.

Part A: Do the following problems from Stewart:

Section 9.1, pages 566-568: exercises 4, 9, 12, 42.

Section 9.2, page 573: exercises 2, 5, 7, 14.

Part B: Do the following problems:

1. Suppose that f is a positive differentiable function. For $x > 0$ let $g(x) = f(x)\sqrt{1 + f'(x)^2}/x$. Let $b > a > 0$. Find the relationship between the following two values:

i) The area of the surface obtained by rotating the graph of $y = f(x)$, from $x = a$ to $x = b$, about the x-axis.

ii) The volume of the solid obtained by rotating the region under the graph of $y = g(x)$, from $x = a$ to $x = b$, about the y-axis.

2. a) Do problem 63 from Section 8.8 of Stewart (page 553). Interpret your answer in terms of the amount of paint that would be needed to fill the interior of this object with paint.

b) Do problem 25 from Section 9.2 of Stewart (page 573). Interpret your answer in terms of the amount of paint that would be needed to paint the outer surface of this object.

c) Explain and resolve the apparent contradiction between your answers to parts (a) and (b).

3. For every real number x, let $f(x) = |x - n|$, where n is the nearest integer to x. If k is a positive integer, let $f_k(x) = f(kx)/k$.

a) Draw the graph of $y = f_k(x)$ for $k = 1, 2, 3, 4$. Find the function g such that $f_k(x) \to g(x)$ as $k \to \infty$.

b) Find the arclength of the graph of $y = f_k(x)$ from $x = 0$ to $x = 1$, for each positive integer k. Also find the arclength of the graph of $y = g(x)$ from $x = 0$ to $x = 1$. Does the former approach the latter as $k \to \infty$?

c) Explain and resolve the apparent contradiction.

4. a) Using the formula for arclength (as an integral), compute the circumference of the circle $x^2 + y^2 = r^2$, and check that this agrees with the known value.

b) Viewing your answer to part (a) as a function of r, note that this is the derivative of the area enclosed by this circle, also viewed as a function of r. Give a geometric explanation for this relationship. [Hint: Use the definition of the derivative as a limit, and find a geometric interpretation for the expression you obtain before taking the limit.]

5. (Three-dimensional analog of problem #4)

a) Let $r > 0$. In terms of r, find a non-negative function f on some interval $a \le x \le b$ such that if the graph of $y = f(x)$ is rotated about the x-axis, then the resulting object is the sphere of radius r centered at the origin.

b) Using a formula for computing a volume of revolution, find the volume enclosed by the sphere.

c) Using a formula for computing a surface area of revolution, find the surface area of the sphere.

d) Do the analog of problem 4(b) in this situation.