Math 104, Lecture 2 Problem Set 6 Spring 2011

Due the week of February 21, 2011, in recitation.

Read Stewart, sections 8.6-8.8.

For each of these two sections, work through the first four core problems, but do not hand them in.

The problems below are to be handed in.

Part A: Do the following problems from Stewart:

Section 8.6, page 529: exercises 5, 9, 11.

Section 8.7, page 541: exercises 7, 9, 20. (For #7 and #9, just write out the summations; you don't have to evaluate them.)

Section 8.8, page 551-552: exercises 11, 23, 34, 49, 51.

Chapter 8 review exercises, page 554 (bottom): exercises 1-6.

Chapter 8 Problems Plus, page 558: exercises 1, 4.

Part B: Do the following problems:

1. Using tables of integrals, find the following:

a) $\int x^2\sqrt{4x^2-1}\,dx$.

b) $\int x\cot^4(x^2)\,dx$.

c) $\int x\arctan(2x)\,dx$.

2. A certain function f has the property that $f(0) = 1$ and $f'(x) = \sin(x^2)$. Estimate $f(\pi)$ to within .01. Explain.

3. Consider the following five methods of approximating definite integrals: left hand rule, right hand rule, midpoint rule, trapezoidal rule, Simpson's rule.

a) In your own words, explain why each of these methods gives an approximation to the value of a definite integral $\int_a^b f(x)\,dx$.

b) If f is an increasing function, which of these methods will systematically give an underestimate? Which will systematically give an overestimate? Why?

c) What happens in part (b) if instead f is a decreasing function?

d) If the graph of f is always concave up, which of these methods will systematically give an underestimate? Which will systematically give an overestimate? Why?

e) What happens in part (d) if instead if the graph of f is always concave down?

4. Consider the five methods of approximation as in problem 3.

a) If $f(x) = Ax + B$ for some real numbers A, B, which of the methods will always give the exact value for $\int_a^b f(x)\,dx$? Why?

b) If $f(x) = Ax^2 + Bx + C$ for some real numbers A, B, C with $A \neq 0$, which of the methods will always give the exact value for $\int_a^b f(x)\,dx$? Why?

5. Determine which of the following improper intervals converge. Explain. [Hint: Do not try to find anti-derivatives.]

a) $\displaystyle\int_0^\infty \cos^{100}(x)\,dx.$

b) $\displaystyle\int_0^\infty \cos^{101}(x)\,dx.$

c) $\displaystyle\int_0^\infty \cos^{100}(x)/x^2\,dx.$

d) $\displaystyle\int_1^\infty \cos(1/x)\,dx.$

e) $\displaystyle\int_1^\infty \sin(1/x)\,dx.$ [Hint: First draw the graphs of $y = \sin(x)$ and of $y = (\sin(1))x$, and see where they meet. Use these graphs to see why $\sin(x) \geq (\sin(1))x$ for $0 \leq x \leq 1$, and therefore why $\sin(1/x) \geq \sin(1)/x$ for $x \geq 1$. Then apply the comparison test to the given integral.]

f) $\displaystyle\int_1^\infty \sin^2(1/x)\,dx.$ [Hint: Explain why $\sin^2(x) \leq x^2$ for $0 \leq x \leq 1$, and therefore why $\sin^2(1/x) \leq 1/x^2$ for $x \geq 1$. Then use the comparison test.]