Math 104, Lecture 2 Problem Set 5 Spring 2011

Due the week of February 14, 2011, in recitation.

Read Stewart, sections 8.4, 8.5.

Work through the first five core problems in section 8.4, and the first four core problems in section 8.5, but do not hand them in. (These are linked from the homework web page, along with the syllabus.)

The problems below are to be handed in.

Part A: Do the following problems from Stewart:

Section 8.4, pages 517-519: exercises 1(a), 3(b), 11, 15, 56-58, 63, 64.
Section 8.5, pages 524-525: exercises 10, 16, 19, 20.

Part B: Do the following problems:

1. A certain differentiable function f has the property that

$$(x^3 + x)f'(x) = 3x^2 - x + 1$$

for all x. Also, $f(1) = 0$. Find $f(1/\sqrt{3})$. [Hint: Find $f'(x)$ and integrate.]

2. a) Describe the possible forms that the partial fraction decomposition may take for a function of the form $f(x)/g(x)$, where f, g are polynomials and g has degree three, depending on the factorization of g.

b) Give examples of functions $f(x), g(x)$ that lead to each of the various possibilities.

c) Relate the various possibilities to the graph of $y = g(x)$, in particular to the way that this graph meets the x-axis. [Hint: This is related to the way that $g(x)$ factors.]

3. Instead of applying partial fractions to rational functions (which are ratios of polynomials), one can apply them to rational numbers (which are ratios of integers), again writing the given expression as a sum of expressions whose denominators are simpler. In doing this, instead of using denominators that are powers of polynomials that cannot be factored (irreducible polynomials, like $x - 3$ and $x^2 + 1$), one uses denominators that are powers of integers that cannot be factored (prime numbers, like 2 and 5). For example, $1/6 = 1/2 - 1/3$ and $7/12 = 1/2^2 + 1/3$. Find partial fraction decompositions for these rational numbers: $1/15$, $1/20$, $17/18$, $13/30$.

4. a) Let $F(x) = \int_0^x e^{t^2}\, dt$. Find $F'(x)$. Explain.

b) Explain why your answer in part (a) does not give a way to evaluate $\int_a^b e^{x^2}\, dx$ explicitly.