Math 104, Lecture 2 Problem Set 3 Spring 2011

Due the week of January 31, 2011, in recitation.

Read Stewart, sections 6.1-6.3, 6.5, and 8.1.

For each of these sections, work through the first four core problems, but do not hand them in. (These are linked from the homework web page, along with the syllabus.)

The problems below are to be handed in.

Part A: Do the following problems from Stewart:

Chapter 6, page 378: Concept check problem 2, and exercises 3, 5, 7, 9, 13, 15(c), 19, 21, 30, 31.
Chapter 8, section 8.1, page 493: exercises 1, 3, 9.

Part B: Do the following problems:

1. Find the areas of each of the following regions:

a) The region above the graph of $y = x^2 - 1$ and below the graph of $y = 1 - x^4$.

b) The region above the graph of $y = \sqrt{|x|}$ and below the line $y = 1$.

c) The region that is below the graph of $y = \sin(x)$ and above the line $y = 1/2$, between $x = 0$ and $x = \pi/2$. [Caution: Be careful about your limits of integration.]

d) The region that is below the graph of $y = \sin(x)$, below the line $y = 1/2$, and above the x-axis, between $x = 0$ and $x = \pi/2$. [Hint: You can use part (c) to obtain a short-cut.]

2. Let f be an increasing differentiable function such that $f(0) = 0$ and $f(1) = 10$. Find the areas of the following regions:

a) The region enclosed between the graphs of $y = f(x)$ and $y = f(x) + e^x$, between $x = 0$ and $x = 1$.

b) The region enclosed between the graph of $y = f(x)$ and the graph of $y = f(x) + \sin(\pi x)$, between $x = 0$ and $x = 1$. [Be careful here.]

c) The region below the graph of $y = f'(x)$ and above the line $y = -1$, between $x = 0$ and $x = 1$.

3. A certain solid object has a flat base and flat top, and is two feet high. At a height of x feet, the horizontal cross section is a square, each of whose sides has length $6 - x^2$ (in feet).

a) Describe what the solid looks like, and draw a picture.

b) Find the volume of the solid.

4. Find the volumes of the following solids of revolution. [You may use either washers or shells, whichever seems better suited to the problem.]

a) The result of rotating around the x-axis the region above the graph of $y = \sin(x)$, below the graph of $y = \sqrt{\sin^2(x) + 1}$, and between $x = 0$ and $x = \pi$.

b) The result of rotating around the y-axis the region above the graph of $y = e^{x^2}$, below the graph of $e^{x^2} + 1$, to the right of the y-axis, and to the left of the line $x = 2$.

5. Let a be a positive real number that is less than 1.

a) Find the average value of the function $1/x$ on the interval $a \leq x \leq 1$. What happens to this average value as $a \to 0^+$?

b) Find the average value of the function $1/\sqrt{x}$ on the interval $a \leq x \leq 1$. What happens to this average value as $a \to 0^+$?

c) Explain geometrically a reason for the quite different answers in parts (a) and (b).

6. Let f be a twice-differentiable function, such that $f(0) = 1$, $f'(0) = 2$, $f(1) = 4$, and $f'(1) = 6$. Evaluate $\displaystyle\int_0^1 x f''(x)\, dx$. [Hint: integrate by parts.]