Math 104, Lecture 2 Problem Set 2 Spring 2011

Due the week of January 24, 2011, in recitation.

Review Stewart, chapters 4, 5, and 7.

Hand in the following problems on the review material:

1. Let $f(x) = x^3 - x$.

a) Find where the function $f(x)$ achieves its maximum and its minimum on the closed interval $[-1, 1]$.

b) Evaluate $\int_{-1}^{1} f(x)\,dx$. How could you have predicted this answer even before computing it?

2. a) Draw the graph of a function f with the following properties: $f(0) = 0$; $f(x) > 0$ for all $x > 0$; $f(x) < 0$ for all $x < 0$; $f'(x) > 0$ for $-1 < x < 1$; $f'(x) < 0$ for $x < -1$ and for $x > 1$; $f''(x) > 0$ for $-2 < x < 0$ and for $x > 2$; $f''(x) < 0$ for $x < -2$ and for $0 < x < 2$.

b) Find all x at which f achieves a maximum or a minimum, and also find all x at which the graph of f has an inflection point.

3. Suppose a certain differentiable function f has the property that $f(0) = 1$ and $f(2) = 5$.

a) Must there be a value of x between 0 and 2 satisfying $f'(x) = 2$? Explain.

b) Must there be a value of x between 0 and 2 satisfying $f'(x) \neq 2$? Or alternatively, is it possible that $f'(x) = 2$ for *all* values of x between 0 and 2? In the latter case, give an example of such a function.

4. Evaluate the following:

a) $\displaystyle\int_0^1 (x^2 + \cos(\pi x/2))\,dx.$

b) $\displaystyle\int \frac{x}{\sqrt[3]{1-2x^2}}\,dx.$

c) $\displaystyle\int_0^1 x^4 \cos(x^5 + 1)\,dx.$

5. Let n be an unspecified positive integer.

a) Find the area of the region that is bounded above by the graph of $y = \sqrt[n]{x}$ and below by the x-axis, between $x = 0$ and $x = 1$.

b) Do the same for the graph of $y = x^n$.

c) Explain geometrically why the answers you got in parts (a) and (b) add up to a particular sum, which doesn't depend on n. (Hint: Treat the regions in (a) and (b) as pieces of a jigsaw puzzle, and move them around.)

6. Find the derivatives of the following functions:

a) e^{e^x}. (Note: a^{b^c} means $a^{(b^c)}$.)

b) $\ln(\ln(x))$.

c) $\ln(e^x)$. (In this part, explain why you got this particular answer.)

d) $\arcsin(2x)$.

e) $\arctan(\sqrt{x})$.

7. Evaluate the following:

a) $\displaystyle\int \frac{1}{1+4x^2}\,dx$. (Hint: See Chapter 7.)

b) $\displaystyle\int \frac{x^2}{1+x^2}\,dx$. (Hint: $\displaystyle\frac{1}{1+x^2} + \frac{x^2}{1+x^2} = \frac{1+x^2}{1+x^2} = 1$.)

c) $\displaystyle\int_0^{2\pi} (\sin x)^{1{,}000{,}001}\,dx$. (Hint: Don't try to evaluate this directly. Instead compare the values of $\int_0^{\pi}$ and $\int_{\pi}^{2\pi}$ geometrically.)

8. Suppose that f is an increasing continuous function, and that $f(0) = 1$ and $f(3) = 6$.

a) Find the largest number m you can such that $\int_0^3 f(x)\,dx \geq m$.

b) Find the smallest number M you can such that $\int_0^3 f(x)\,dx \leq M$.

c) Is it possible that $\int_0^3 f(x)\,dx = m$ for some choice of f? Is it possible that $\int_0^3 f(x)\,dx = M$ for some choice of f? Explain.

9. a) Evaluate $\int \sin(x)\cos(x)\,dx$ in each of the following three ways:

i) Use the substitution $u = \cos(x)$.

ii) Use the substitution $u = \sin(x)$.

iii) Use the double angle formula $\sin(2x) = 2\sin(x)\cos(x)$.

b) Explain how it can be that you got different answers from the different methods. Are all of them correct? Are any of them correct? What is the actual solution to $\int \sin(x)\cos(x)\,dx$?

10. Evaluate the following:

a) $\displaystyle\lim_{x\to 0} \frac{\sin x}{e^x - 1}$.

b) $\displaystyle\lim_{x\to \pi} \frac{\sin x}{e^x - 1}$.

c) $\displaystyle\lim_{x\to 0^+} xe^{1/x}$.