Math 609 / AMCS 609: Real Analysis
Spring 2012
TTh 3:00–4:30, DRL 4C8

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Office Hours:
Thursday 4:30–5:30
or by appointment

Course Description: This year we will reverse the roles of math 608 and 609 and combining with the corresponding sequence in the AMCS program. The official course descriptions to be used as a guide are as follows:

> **MATH 609** Lebesgue measure and integral, Borel measures, convergence theorems. Banach spaces, Hahn-Banach Theorem, Lp-spaces, Riesz-Fischer theorem, Stone-Weierstrass theorem, Radon-Nikodym theorem. Applications to Fourier series and integrals, Plancherel Theorem, Distributions, convolutions and mollifiers. Partitions of unity. Applications to P.D.E.'s
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> **AMCS 609** ...We then turn to analysis in infinite dimensional spaces. After covering the basic results on complete normed linear spaces, we discuss the Riesz-Fredholm theory of integral equations. Second semester: We consider Hilbert space, unbounded operators, self adjointness and the spectral theorem, with applications to ordinary differential equations. After a discussion of the basic Sobolev and Holder spaces we show how to use classical and functional analytic techniques to solve partial differential equations. The course concludes with a discussion of numerical techniques for the solution of ODES and PDES and their mathematical foundations.

This course will largely be self-contained, but for many it will be an unsuitable place to begin. You must have taken 508/509 or some equivalent:

> **MATH 508** Construction of the real numbers, the topology of the real line and the foundations of single variable calculus. Notions of convergence for sequences of functions. Basic approximation theorems for continuous functions and rigorous treatment of elementary transcendental functions. The course is intended to teach students how to read and construct rigorous formal proofs. A more theoretical course than Math 360.
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> **MATH 509** Continuation of Math 508. The Arzela-Ascoli theorem. Introduction to the topology of metric spaces with an emphasis on higher dimensional Euclidean spaces. The contraction mapping principle. Inverse and implicit function theorems. Rigorous treatment of higher dimensional differential calculus. Introduction to Fourier analysis and asymptotic methods.

Regarding complex analysis: There will be occasions in which knowledge of the Cauchy theory of single variable complex functions will be necessary.

Useful Books: Given the wide breadth of material we may cover this semester, recommending texts is particularly difficult. If you were to purchase only one book, I would recommend Real Analysis: Modern Techniques and Their Applications by Gerald B. Folland, published by Wiley-Interscience. It covers nearly all of the topics mentioned above; the chief complaints you will hear about this book are that it falls a little on the abstract side and it's somewhat dry. If you've got the cash, I would also pick up Measure and Integral: An Introduction to Real Analysis by Richard L. Wheeden and Antoni Zygmund, published by Marcel Dekker, Inc. It's a nice complement to Folland; it covers less material but it's more concrete.

If you're a big Stein and Shakarchi fan, you can also go that route (though they do things in a somewhat non-standard order, so you may sometimes feel that the correspondence between lecture and text is a little stretched). The relevant books are Real Analysis: Measure Theory, Integration, and Hilbert Spaces, Functional Analysis: Introduction to Further Topics in Analysis, and Fourier Analysis: An Introduction. All are published by Princeton University Press. Real Analysis is clearly the first one to buy if you're on a budget; after that it's basically a tie for second place.

Course Website: <https://courseweb.library.upenn.edu/> (coming online soon). Homework assignments will be posted on a weekly basis, usually on Thursday.

Grade System: Homework will be worth 30% of the grade. Weekly assignments are due in Ted Spaide's mailbox in the math office before 3pm on Friday. Tentatively, there will be a take-home midterm due **Wednesday, February 29th** which is worth 30% of the grade as well. The final exam will (likely) be a take-home exam due **Wednesday, May 2nd** at 12:00; it will be worth the remaining 40% of the grade.