CALCULUS BLUE SYLLABUS

PART 1: VECTORS & MATRICES

1. **Lines and planes in R^3**
2. **Curves and surfaces in R^3**
3. **Coordinates in R^n**
   -applications: robot kinematics, wireless signals, customer profiles
4. **Vectors**
5. **The dot product**
   -application: matching methods for dating services
6. **The cross product**
   -also, the scalar triple product
7. **Intro to vector calculus**
   -application: celestial mechanics
8. **Vector calculus & motion**
   -application: curvature, tangential & normal acceleration
9. **Matrices**
   -applications: image processing, genetic correlations, predictive text, …
10. **Matrix algebra**
11. **Matrices and equations**
    -applications: balancing chemical equations, network traffic flow, commodity management
12. **Row reduction**
13. **Inverse matrices**
14. **Linear transformations**
    -application: Euler angles & 3-d rotations in robotics & graphics
15. **Coordinate transformations**
16. **Determinants: algebraic**
17. **Determinants: geometric**
18. **Determinants: algorithmic**
    -application: computational complexity of determinants

PART 2 : DIFFERENTIATION

1. **Multivariate functions**
   -applications: meteorology, robotics, SEIR models, market equilibria, …

2. **Rates of change**
   -application: changing commodity prices
3. **Definition of the derivative**
   -using linear transformations
4. **Differentiation**
   -formal definition ; interpretation as first order variation
5. **The chain rule**
   -via matrix multiplication / composition of linear transformations
6. **Derivative rules**
   -application: the material derivative
7. **The Inverse Function Theorem**
   -applications: solving systems of nonlinear equations locally; inverse kinematics
8. **The Implicit Function Theorem**
   -application: GPS sensitivity
9. **Gradients**
   -application: machine learning
10. **Tangent lines/planes**
11. **Linearization & approximation**
    -application: numerical approximation; tolerances and error estimation in engineering
12. **Taylor expansions**
13. **Computing Taylor expansions**
14. **Critical points and Hessians**
15. **Optimization: Linear regression**
    -applications: statistics
16. **Optimization: Game theory [OPTIONAL]**
    -application: Nash equilibria
17. **Constrained optimization**
18. **The Lagrange multiplier**
    -application: shadow prices
19. **Lagrange examples**
    -applications: stock supply management, AMGM inequality

PART 3 : INTEGRATION

1. **Definitions of Integrals**
2. **The Fubini Theorem**
3. **Double integrals**
4. **Triple integrals**

5. **Averages**
   -application: L^2 (RMS) averages
6. **Mass & centers**
   -application: monostatic solids
7. **Moments of inertia**
8. **The Inertia matrix**
9. **Solid body mechanics**
   -applications: angular velocity, momentum, kinetic energy, etc.
10. **Review of probability**
11. **Multiple random variables**
    -application: portfolio management
12. **Covariance matrices [OPTIONAL]**
    -application: target tracking and uncertainty modeling
13. **Polar & cylindrical coordinates**
    -application: Gaussians
14. **Spherical coordinates**
15. **The Change of Variables Theorem**
    -in all dimensions, using determinant of derivative
16. **Changing coordinates**
    -application: computing work in thermodynamics
17. **Surface integrals**
18. **Gaussians, redux [OPTIONAL]**
    -application: the Kalman filter and data fusion
19. **Spheres and Data [OPTIONAL]**
    -application: the Gaussian Annulus Theorem

PART 4 : CALCULUS OF FIELDS

1. **Fields**
2. **Scalar Path integrals**
3. **Integrating 1-forms**
4. **The Independence of Path Theorem**
5. **Work, Circulation, & Flux**
   -application: circulation and flux in fluids
6. **Green's Theorem**
7. **Euclidean forms in 3-d**
8. **Grad, Curl, & Div**

9. **Integrating 2-forms**
10. **Gauss's Theorem in 3-d**
11. **Stokes' Theorem in 3-d**
12. **When to use Which Theorem**
13. **Forms & Fluids [OPTIONAL]**
    -application: Kelvin Circulation Theorem & Helmholtz Theorem
14. **Forms & Physics [OPTIONAL]**
    -applications: Maxwell's equations
15. **Forms & Data**
    -applications: medical imaging, geodesy
16. **Differential forms [OPTIONAL]**
17. **Calculus of forms [OPTIONAL]**
    -application: geometric optics
18. **Stokes' Theorem Redux [OPTIONAL]**
    -application: time-series data