Math 103: L'Hopital's Rule

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For some limits evaluation via substitution gives meaningless expressions called **Indeterminate Forms**

$$lim_{x \to 0} \frac{sin(x)}{x} = \frac{0}{0}$$

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Image: A matrix

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Other indeterminate forms include ∞ : 0, 0, and 1 , and 1

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L'Hopital's Rule for $\frac{0}{0}$

Theorem

Suppose f(a) = g(a) = 0, f and g are differentiable near a and $g'(x) \neq 0$ for x near a but not equal to a, Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

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Image: Image:

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This only helps us with indeterminate forms $\frac{0}{0}$.

L'Hopital's Rule for $\frac{\infty}{\infty}$

Theorem

Suppose $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to a$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

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The theorem also holds for one-sided limits and infinite limits.

Must convert other indeterminate forms to $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

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- Find where the graph of f is increasing and decreasing.

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- Find where the graph of *f* is increasing and decreasing.
- Find the points of inflection and the concavity of f.
- Identify any asymptotes.
- Plot key points and asymptotes, and sketch the curve.

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