Math 103: Trig Derivatives and Rate of Change Problems

Ron Donagi

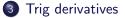
University of Pennsylvania

Thursday February 9, 2012









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Review

Derivative Rules

•
$$\frac{d}{dx}(c) = 0$$

• $\frac{d}{dx}(x^n) = nx^{n-1}$
• $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$
• $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$
• $\frac{d}{dx}[a^x] = ln(a)a^x$
• $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$
• $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$

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Instantaneous Velocity

Definition

If s(t) is a position function defined in terms of time t, then the instantaneous velocity at time t = a is given by $v(a) = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$

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ExampleSuppose a penny is dropped from the top of DRL which is 19.6 meters high. The position of the penny in terms of hight above the street is given by $s(t) = 19.6 - 4.9t^2$. At what is the velocity of the penny when it hits the ground.

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Position, Velocity, Acceleration and Jerk

If the position of a body at time t is given by s(t) then

- Velocity at time t is given by $v(t) = \frac{ds}{dt}$
- Acceleration at time t is given by $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
- Jerk at time t is given by $j(t) = \frac{da}{dt} = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}$

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$$s(t)=-3cos(t)$$

What is the velocity and acceleration of the weight at time *t*?

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•
$$\lim_{\theta \to 0} \frac{\sin(\theta) - \sin(0)}{\theta - 0} = \lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$$

• $\lim_{\theta \to 0} \frac{(\cos(\theta) - \cos(0))}{\theta - 0} = \lim_{\theta \to 0} \frac{(\cos(\theta) - 1)}{\theta} = 0$

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- $\frac{d}{dx}(\cos(x)) = -\sin(x)$
- $\frac{d}{dx}(tan(x)) = (sec(x))^2$

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• $\frac{d}{dx}(cos(x)) = -sin(x)$ • $\frac{d}{dx}(tan(x)) = (sec(x))^2$ • $\frac{d}{dx}(csc(x)) = -csc(x)cot(x)$

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Trig derivatives

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