Math 103: Derivatives of Inverse Functions and Logs

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Outline

Derivatives of Inverse Functions

Suppose $f(x)$ is a function with inverse $f^{-1}(x)$ with each defined on the appropriate domain and range.

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Exercise Find the the slope of the tangent line to $y = x^2$ at $(2, 4)$ and find the slope of the tangent line to $y = \sqrt(x)$ at $(4, 2)$.

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4. $\frac{d}{dx}(ln(x)) = \frac{1}{x}$

Change of Base Formula

For any positive number a ($a \neq 0$), we have

$$log_a(x) = \frac{ln(x)}{ln(a)}$$

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6. $\frac{d}{dx}(csc^{-1}(x)) = \frac{-1}{|x|\sqrt{x^2-1}}$