Math 103: One-Sided Limits of Functions

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Outline

Definition of Limit

Definition

If $f(x)$ is arbitrarily close to L for all x sufficiently close to x_0, we say f approaches the **limit** L as x approaches x_0 and write:

$$lim_{x \to x_0} f(x) = L$$

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Last time we saw

1. Limit laws
2. Theorems regarding polynomials and rational functions
3. How to evaluate a limit if there is a zero in the denominator

The Sandwich Theorem

Theorem

If $f(x) \leq g(x) \leq h(x)$ *when* x *is near* c *and*

$$lim_{x \to c} f(x) = lim_{x \to c} h(x) = L$$

$$then \ lim_{x \to c} g(x) = L$$

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Evaluate:

$$lim_{x \to 0} \ x^2 sin(\frac{1}{x})$$

Definition of One-Sided Limit

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If $f(x)$ is arbitrarily close to L for all x sufficiently close to c and greater than c, we say f approaches the **rigth-hand limit** L as x approaches c and write:
$lim_{x \to c^+} f(x) = L$

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If $f(x)$ is arbitrarily close to L for all x sufficiently close to c and greater than c, we say f approaches the **rigth-hand limit** L as x approaches c and write:
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Definition

If $f(x)$ is arbitrarily close to L for all x sufficiently close to c and less than c, we say f approaches the **left-hand limit** L as x approaches c and write:
$lim_{x \to c^-} f(x) = L$

Theorem

$$lim_{x \to c} f(x) = L$$

if and only if

$$lim_{x \to c^+} f(x) = L \text{ and } lim_{x \to c^-} f(x) = L.$$

Theorem

$$lim_{x \to 0} \frac{sin(x)}{x} = 1$$