KEY

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1. Evaluate the following limit:

$$\lim_{x\to 0}\frac{\cos x-\cos 5x}{\sin x}$$

Observe that $\lim_{x\to 0}(\cos x-\cos 5x)=\cos(0)-\cos(0)=0$ and $\lim_{x\to 0}\sin x=\sin(0)=0$.

Therefore, this limit is of type $\frac{0}{0}$ and we may use L'Hospital's Rule.

$$\begin{aligned}\lim_{x\to 0}\frac{\cos x-\cos 5x}{\sin x}&=\lim_{x\to 0}\frac{-\sin x+5\sin 5x}{\cos x}\\&=\frac{-\sin(0)+5\sin(0)}{\cos(0)}\\&=\frac{0}{1}=0\end{aligned}$$

2. Find the derivative of $f(x)=\cosh(x)\sinh(x)$.

Method 1:

Recall that the derivative of $\cosh x$ is $\sinh x$ and the derivative of $\sinh x$ is $\cosh x$.

Using product rule, we obtain:

$$\begin{aligned}f'(x)&=\frac{d}{dx}[\cosh x]\sinh x+\cosh x\frac{d}{dx}[\sinh x]\\&=\sinh^2 x+\cosh^2 x.\end{aligned}$$

Method 2:

Recall that $\sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$, so in particular, $\sinh(2x)=2\sinh x\cosh x$.

Therefore, $f(x)=\frac{1}{2}\sinh(2x)$.

So, $f'(x)=\frac{1}{2}2\cosh(2x)=\cosh(2x)$.