Math 103-920 Quiz 4

KEY

Show your work to receive full credit!

1. Find the local minima and maxima, and inflection points of the function

$$f(x) = x^3 - \frac{3}{2}x^2 - 12x + 5$$

$$f'(x) = 3x^2 - 3x - 12 = 3(x^2 - x - 4)$$
$$f''(x) = 6x - 3 = 3(2x - 1)$$

$f'(x) = 0$ when $x = \dfrac{1 \pm \sqrt{17}}{2}$, and $f''(x) = 0$ when $x = 1/2$.

Since $f''\left(\dfrac{1-\sqrt{17}}{2}\right) < 0$, this point is a local maximum.

Since $f''\left(\dfrac{1+\sqrt{17}}{2}\right) > 0$, this point is a local minimum.

Since $f''(x)$ changes sign at $x = 1/2$, this is an inflection point.

2. For the function $f(x)$ above, find its absolute minimum and maximum on the interval $[-5, 5]$.

We check $f(x)$ at the endpoints and at critical numbers.

$$f(-5) = -97.5$$
$$f(5) = 32.5$$
$$f\left(\frac{1-\sqrt{17}}{2}\right) = \frac{17\sqrt{7}-5}{4} \approx 16.27$$
$$f\left(\frac{1+\sqrt{17}}{2}\right) = \frac{-17\sqrt{7}-5}{4} \approx -18.77$$

So the maximum value is 32.5 achieved at $x = 5$, and the minimum is -97.5 achieved at $x = -5$.