Homework Set 1

1. Chapter 1 of do Carmo: 1,2,3,4,5,7.

2. Chapter 2 of do Carmo: 1,2,3,4,5,8.

3. Let \((M^n, g)\) be a Riemannian manifold and \(d\) the induced Riemannian distance. Prove that \((M, d)\) is a metric space and that the induced metric topology agrees with the manifold topology.

4. Show that all geodesics of a Lens space (see Homework 0) are closed.

5. Find a closed manifold which contains a geodesic which is not closed but which is not dense. (Contrarily to what happens on the flat two-torus). Hint: You can inspire yourself on Problem 1 of Chapter 2 of Carmo.)