Math 240, Fall 2019 **Davi Maximo**

Homework Set 2

DUE: SEP 17, 2018 (BEGINNING OF CLASS)

DO but DO NOT HAND IN the following problems from Goode and Annin:

1. Section 2.5, True and False.
2. Section 2.6, True and False.

DO and SUBMIT the following problems from Goode and Annin:

3. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 3 & 7 \end{pmatrix}$.
   (a) Show that A and B don't commute, i.e. that $AB \neq BA$.
   (b) Compute the matrix $(A+B)^2$.
   (c) Compute the matrix $A^2 + 2AB + B^2$.
   (d) Notice that your answers in part (b) and part (c) are different, i.e. apparently the familiar algebraic identity doesn't seem to hold for square matrices. Explain, in general, why this is the case, i.e. why in general $(A+B)^2 \neq A^2 + 2AB + B^2$. What conditions should the square matrices A and B satisfy for the identity to hold?
4. Section 2.5, 6-7.
5. Determine whether the following statements are true or false. Justify your answer.
   (a) Let A be an $n \times n$ matrix such that $A^2 = 0$. Then the matrix $I_n - A$ is invertible with inverse $I_n + A$. (Here I_n denotes the $n \times n$ identity matrix.)
   (b) The only $n \times n$ matrices whose inverse is itself are I_n and $-I_n$.
   (c) If a square matrix A is invertible, then its transpose A^T is also invertible.
   (d) The inverse A^{-1} of an invertible symmetric matrix A is also symmetric.
   (e) The product of two 2019×2019 matrices with rank 2019 is also of rank 2019.
   (f) The sum of two invertible matrices is also invertible.
   (g) If the $n \times n$ matrix A is not invertible, then the linear system $A\vec{x} = \vec{b}$ has infinitely many solutions for every $\vec{b} \in \mathbb{R}^n$.
   (h) If the $n \times n$ matrix A in not invertible, then the homogenous linear system $A\vec{x} = 0$ has infinitely many solutions.
6. Section 2.6, 3-4.
7. Section 2.6, 9-10.

8. Section 2.6, 17.
9. Section 2.6, 41.
10. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$.
    (a) Compute the inverse A^{-1}.
    (b) Use your answer in part (a) to find the unique solution to the linear system $A\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.
11. If $n \times n$ matrices A and B are invertible and $AB = 0$, show that either $A = 0$ or $B = 0$.
12. Spring 2016 Final, Problem 4. See: `https://www.math.upenn.edu/ugrad/calc/m240/exams/240spring16final.pdf`

Extra Credit Problems

13. Goode and Annin, Section 2.6, 44.
14. Goode and Annin, Section 2.6, 39-40.
15. Problem 11 of *Some more challenging linear algebra problems*: `https://www.math.upenn.edu/sites/www.math.upenn.edu/files/240la.pdf`