

**MATH 360 –Homework 11**  
Due Monday, December 11, 2017

**To discuss in recitation:**

1. Let  $K$  be a real function of  $x$  such that  $K \geq 0$ ,  $K$  is continuous, and zero outside a bounded interval, and

$$\int_{-\infty}^{\infty} K(t) dt = 1.$$

Define  $K_n(t) = nK(nt)$  Show that  $\{K_n\}$  is a Dirac sequence (approximate identity).

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2. Now suppose that  $K$  is infinitely differentiable, and such that  $K(t) = 0$  if  $t$  is outside some bounded interval. Suppose  $f$  is a bounded continuous function. Show that  $K * f$  is infinitely differentiable and that  $(K * f)' = K' * f$ .
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3. Textbook page 321, problems 1, 2, 5, 10
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4. Textbook page 327, problems 1, 6, 11, 14
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5. Textbook page 336, problems 2, 5, 7
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**To be handed in on December 11:**

1. Let  $f(x)$  be continuous on  $[0, 1]$ . Suppose

$$\int_0^1 x^n f(x) dx = 0$$

for every whole number  $n = 0, 1, 2, 3, \dots$ . Prove that  $f = 0$ . (*Hint*: Weierstrass approximation theorem – approximate  $f$  by polynomials and show that the integral of  $f^2$  is zero; then refer to that problem on the second midterm).

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2. Prove that if  $f$  is a continuous function, then

$$\lim_{h \rightarrow 0^+} \int_{-1}^1 \frac{h}{h^2 + x^2} f(x) dx = \pi f(0).$$

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3. Textbook page 321, problems 3, 4, 8, 11
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4. Textbook page 327, problems 2, 9, 17
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5. Textbook page 337, problems 8, 10, 11