University of Pennsylvania
Math 104 Section 3
Fall 2016

Name: _____

Penn ID#: _____

Midterm Examination #3

This exam contains eight questions. No calculators are allowed, but you may use one standard sized 8.5"x11" sheet with notes handwritten on both sides. Show your work in the space provided below, and circle your final answer. Little or no credit will be given for an answer with no supporting work, even if correct. If you change an answer, please either erase or cross out the answer you do not want to be considered.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination. In particular, all the work on this test is my own.

_____
Signature

1. Solve the differential equation $\frac{dy}{dx} - 2xy = x$.

(A) $y = \frac{1}{2}x^2 - C$

(B) $y = -\frac{1}{2}x^2 - C$

(C) $y = Ce^{-x^2} - \frac{1}{2}$

(D) $y = Ce^{x^2} - \frac{1}{2}$

(E) $y = -2x + e^{x^2} - C$

(F) $y = x + e^{x^2} - C$

(G) $y = e^{x^2} + \frac{1}{2} + C$

(H) $y = e^{-x^2} - \frac{1}{2} + C$

2. Determine if the sequence $a_n = (-1)^n \frac{n^2}{1+n^3}$ converges or diverges. If the sequence converges, find its limit.

(A) Divergent, $\lim_{n\to\infty} a_n = 0$ (B) Convergent, $\lim_{n\to\infty} a_n = 1$

(C) Convergent, $\lim_{n\to\infty} a_n = 0$ (D) Convergent, $\lim_{n\to\infty} a_n = -1$

(E) Convergent, $\lim_{n\to\infty} a_n = 3$ (F) Divergent, $\lim_{n\to\infty} a_n = \infty$

3. Determine if the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}}$ converges or diverges. If the series converges, find its sum.

(A) Convergent, $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}} \to 0$ (B) Divergent, $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}} \to \infty$

(C) Divergent, $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}} \to 0$ (D) Convergent, $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}} \to -\frac{1}{2}$

(F) Convergent, $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}} \to 1$ (G) Convergent, $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}} \to \frac{1}{2}$

4. Does the series $\sum_{n=1}^{\infty}\left(\frac{\ln n}{n}\right)^2$ converge or diverge?

(A) Converge (B) Diverge

5. Determine whether the following series converge or diverge:

I. $\sum_{n=1}^{\infty}\frac{3+\cos n}{3^n}$ II. $\sum_{n=1}^{\infty}\frac{1+2^n}{1+3^n}$

(A) I & II both divergent (B) I convergent & II divergent

(C) I divergent & II convergent (D) I & II both convergent

6. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent:

I. $\sum_{n=1}^{\infty} \frac{(n+2)!}{n!10^n}$ II. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+4}$

(A) I divergent; II absolutely convergent

(B) I absolutely convergent; II divergent

(C) I conditionally convergent; II conditionally convergent

(D) I divergent; II conditionally convergent

(E) I absolutely convergent; II conditionally convergent

(F) I absolutely convergent; II absolutely convergent

7. Find the radius (R) and interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{3^n x^n}{(n+1)^2}$.

(A) Diverges (B) $R = \frac{1}{3}, \left[-\frac{1}{3}, \frac{1}{3}\right]$ (C) $R = \frac{1}{3}, \left(-\frac{1}{3}, \frac{1}{3}\right]$

(D) $R = 3, [-3, 3)$ (E) $R = \frac{1}{3}, \left(-\frac{1}{3}, \frac{1}{3}\right)$ (F) $R = 1, [-3, 3]$

8. Find the Maclaurin series for $f(x) = \cos(x^3)$ and its radius of convergence (R).

(A) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6(2n+1)}}{(2n+1)!}, R = \infty$ (B) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{(2n)!}, R = \infty$

(C) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}, R = \infty$ (D) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}, R = 1$

(E) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, R = 0$ (F) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{(2n)!}, R = \infty$