

MATH 104 – First Midterm Exam - Fall 2015

1. Find the area between the graphs of $y = 9 - 2x$ and $y = x^2 + 1$.

- (a) 24 (b) 36 (c) $\frac{32}{3}$ (d) $\frac{256}{3}$ (e) $\frac{4}{3}$ (f) $\frac{81}{2}$
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2. A solid has as its base the region in the xy -plane the region between the curve $y = 1 - \frac{x^2}{4}$ and the x -axis. The cross-sections of the solid perpendicular to the y -axis are squares with one side in the base. What is the volume of the solid?

- (a) 72 (b) 144 (c) 18 (d) 32 (e) 24 (f) 8
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3. Let $V(b)$ be the volume of the solid obtained by rotating the area between the graph of $y = e^{-4x^2}$ and the x -axis for $0 < x < b$ around the y -axis.

What is $\lim_{b \rightarrow \infty} V(b)$?

- (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$ (e) $\frac{\pi}{6}$ (f) $\frac{\pi}{8}$
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4. Find the length of the part of the curve $y = e^x + \frac{1}{4}e^{-x}$ for $0 \leq x \leq \ln 2$.

- (a) $\frac{19}{12}$ (b) $\frac{11}{12}$ (c) $\frac{3}{8}$ (d) $\frac{9}{8}$ (e) $\frac{29}{12}$ (f) $\frac{3}{4}$
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5. $\int_{\frac{1}{5}}^{\frac{1}{5}e^4} \frac{\ln(5x)}{x} dx$

- (a) 5 (b) 14 (c) 32 (d) 72 (e) 8 (f) 18
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MATH 104 – First Midterm Exam - Fall 2014

1. Find the area between the graphs of $y = x^2 + 3x + 5$ and $y = 2x^2 - x$.

(a) 24 (b) 36 (c) $\frac{343}{6}$ (d) $\frac{125}{6}$ (e) $\frac{32}{3}$ (f) $\frac{81}{2}$

2. A solid has as its base the region in the xy -plane the region between the curve $y = 1 - \frac{x^4}{81}$ and the x -axis. The cross-sections of the solid perpendicular to the x -axis are semicircles with the flat side (i.e., the diameter) in the base. What is the volume of the solid?

(a) $\frac{8\pi}{15}$ (b) $\frac{2\pi}{15}$ (c) $\frac{2\pi}{5}$ (d) $\frac{16\pi}{45}$ (e) $\frac{8\pi}{45}$ (f) $\frac{4\pi}{15}$

- 3.** Let $V(a)$ be the volume of the solid obtained by rotating the area between the graph of $y = \frac{1}{x^6}$ and the x -axis for $1 < x < a$ around the y -axis.

What is $\lim_{a \rightarrow \infty} V(a)$?

(a) 4π (b) 2π (c) $\frac{3\pi}{2}$ (d) π (e) $\frac{2\pi}{3}$ (f) $\frac{\pi}{2}$

4. Find the length of the part of the curve $y = \frac{1}{12}x^3 + \frac{1}{x}$ for $1 \leq x \leq 2$.

(a) $\frac{17}{12}$ (b) $\frac{13}{12}$ (c) $\frac{19}{12}$ (d) $\frac{23}{12}$ (e) $\frac{29}{12}$ (f) $\frac{43}{12}$

5. $\int_0^{\pi/6} \frac{\cos(3x)}{1 + \sin^2(3x)} dx$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{12}$ (e) $\frac{\pi}{16}$ (f) $\frac{\pi}{24}$

6. An object moves in such a way that its acceleration is $6e^{-t/3}$ meters per second². If the initial velocity of the object is 5 meters per second, what is the limit of its velocity as $t \rightarrow \infty$?

- (a) 9 m/s (b) 13 m/s (c) 14 m/s
(d) 16 m/s (e) 17 m/s (f) 23 m/s
-

7. Find the surface area obtained by rotating the part of the curve $y = x^2 + 1$ for $0 \leq x \leq 1$ around the y -axis.

- (a) $\frac{\pi}{6}(5\sqrt{5} - 1)$ (b) $\frac{\pi}{6}(17\sqrt{17} - 1)$ (c) $\frac{\pi}{6}(37\sqrt{37} - 1)$
(d) $\frac{\pi}{6}(65\sqrt{65} - 1)$ (e) $\frac{\pi}{6}(82\sqrt{82} - 1)$ (f) $\frac{\pi}{6}(145\sqrt{145} - 1)$
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8. Find the volume obtained by rotating the region between the curve $y = \sqrt{1 - 4x^2}$ and the x -axis around the x -axis.

- (a) $\frac{2\pi}{9}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{4\pi}{9}$ (e) $\frac{2\pi}{3}$ (f) $\frac{\pi}{2}$
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9. The region inside the triangle with vertices $(1, 0)$, $(1, 1)$ and $(0, 0)$ is rotated around the line $x = -5$. What is the volume of the resulting solid?

- (a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{3}$ (c) $\frac{8\pi}{3}$ (d) $\frac{11\pi}{3}$ (e) $\frac{14\pi}{3}$ (f) $\frac{17\pi}{3}$
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10. The region between the x -axis, the y -axis, and the line $y = 8 - 8x$ is revolved around the y -axis to generate a cone. The centroid (center of mass assuming constant density) of this cone is on the y -axis. What is its y -coordinate?

(*Hint:* Use disks. What is the center of mass of a disk? What fraction of the total mass is in each disk? It will save an integral to know that the volume of a cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the base of the cone and h is its height.)

- (a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 1 (e) 2 (f) 4
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Forty more practice problems

1. $\int_e^{e^4} \frac{1}{x\sqrt{\ln x}} dx =$
(a) 0 (b) 2 (c) 4 (d) e^2 (e) $e^4 - e$

2. We know that the graph of a function $\varphi(x)$ passes through the point $(4,6)$. We also know that $-1 \leq \varphi'(x) \leq 3$ for all x in the interval $[0, 10]$. What are the minimum and maximum values possible values for $\varphi(7)$?
(a) min = 7, max = 16 (b) min = 4, max = 7 (c) min = -1, max = 4
(d) min = 6, max = 10 (e) min = 4, max = 13 (f) min = 3, max = 15
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3. Find the area between the graphs of $y = 4x + 5$ and $y = x^2$.
(a) 24 (b) 36 (c) $\frac{343}{6}$ (d) $\frac{125}{6}$ (e) $\frac{32}{3}$ (f) $\frac{81}{2}$
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4. What is the volume of the solid obtained by rotating the part of the graph of $y = \cos x \sqrt{\sin x}$ between $x = 0$ and $x = \frac{\pi}{2}$ around the x -axis?
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$
(d) $\frac{\pi}{4}$ (e) $\frac{\pi}{6}$
-

5. $\lim_{b \rightarrow \infty} \int_0^b \frac{\arctan x}{1 + x^2} dx =$
(a) $\frac{\pi^2}{2}$ (b) π^2 (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{8}$ (e) $+\infty$

6. The area between a curve $y = f(x)$ and the x -axis between $x = 1$ and $x = b$ is rotated around the x axis and the volume of the resulting solid is $\frac{\pi}{3}(\ln b)^3$, for all $b > 1$. What is $f(x)$?
(a) $(\ln x)^2$ (b) $\frac{\ln x}{\sqrt{x}}$ (c) $\frac{\ln x}{x}$ (d) $\frac{(\ln x)^2}{x}$ (e) $\frac{(\ln x)^2}{\sqrt{x}}$
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7. $\int_0^\pi \sin^2(2x) dx =$
 (a) 0 (b) 1 (c) $1/2$ (d) $\pi/2$ (e) π (f) 2π

8. Suppose that $f(x)$ and $g(x)$ are two differentiable functions, and we know the following values:

$$f(0) = 2, \quad f(1) = 3, \quad g(0) = 1, \quad g(1) = 4.$$

Then $\int_0^1 (f(x)^2 g'(x) + 2f(x)f'(x)g(x)) dx =$

(a) 6 (b) 12 (c) 14 (d) 16 (e) 32 (f) 48

9. The area bounded by the curves $y = x^2 - 1$ and $y = 2x + 7$ is

(a) 4 (b) 9 (c) 16 (d) 24 (e) 30 (f) 36

10. If $f(x)$ is continuous for all x and $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 4$, then

(a) $f(0) = 1$ and $f'(0) = 4$ (b) $f(0) = 0$ and $f'(0) = 4$
 (c) $f(0) = 4$ and $f'(0) = 0$ (d) $f(0) = 0$ and $f'(0) = 1$
 (e) $f(0) = 0$ and $f'(0) = 0$ (f) $f(0) = 4$ and $f'(0) = 1$

11. $\frac{d}{dx} \int_0^{\sin x} e^{t^2} dt =$

(a) $e^{\sin^2 x}$ (b) $\cos x e^{\sin^2 x}$ (c) $2 \sin x \cos x e^{\sin^2 x}$
 (d) $\cos x + e^{\sin^2 x}$ (e) $2 \sin x e^{\sin^2 x} + e^{\sin^2 x}$ (f) $e^{\cos^2 x}$

12. $\int_{1/e}^e \frac{(\ln x)^2}{x} dx =$

(a) $\frac{e^2}{2} - \frac{2}{e^2}$ (b) $\frac{e}{2} - \frac{2}{e}$ (c) $\frac{2e}{3}$
 (d) $\frac{2}{3}$ (e) $\frac{e^2}{2} - \frac{1}{2e^2}$ (f) $\frac{2e^2}{3}$

13. What is the average value of the function $f(x) = \sqrt{\frac{\arcsin x}{1-x^2}}$ on the interval $[0, \frac{1}{2}]$?

(a) $\frac{\pi\sqrt{6}}{15}$ (b) $\frac{\pi^{3/2}\sqrt{2}}{54}$ (c) $\frac{\pi\sqrt{3}}{18}$ (d) $\frac{\pi^{3/2}\sqrt{6}}{27}$ (e) $\frac{\pi\sqrt{2}}{48}$

14. What is the volume of the solid obtained by rotating the region bounded by the graphs of $y = \sqrt{x}$, $y = 2 - x$ and $y = 0$ around the x -axis?

(a) $\frac{5\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{2\pi}{5}$ (d) $\frac{11\pi}{6}$ (e) $\frac{5\pi}{3}$ (f) $\frac{7\pi}{6}$

15. The functions $\cosh x$ and $\sinh x$ are defined as $\cosh x = \frac{1}{2}(e^x + e^{-x})$ and $\sinh x = \frac{1}{2}(e^x - e^{-x})$. Calculate the area bounded by the curves $y = \cosh x$, $y = \sinh x$, $x = 0$ and $x = b$. What is the limit of this area as $b \rightarrow \infty$?

(a) 1 (b) e (c) $e + \frac{1}{e}$ (d) $\frac{1}{e}$ (e) ∞

16. Suppose $f(x)$ is a monotonically increasing function on the interval $[0, \infty)$, and that $f(0) = 2$ and $\lim_{x \rightarrow \infty} f(x) = 6$. Suppose $A(b)$ is the average value of $f(x)$ on the interval $[0, b]$ for $b > 0$. What is $\lim_{b \rightarrow \infty} A(b)$?

(a) 2 (b) 4 (c) 5
(d) 6 (e) 0 (f) ∞

17. What is the volume obtained by revolving the region bounded by $y = x^2 - 4$ and $y = 4 - x^2$ around the line $x = 2$?

(a) $\frac{16\pi}{3}$ (b) $\frac{64\pi}{3}$ (c) $\frac{256\pi}{3}$ (d) $\frac{512\pi}{3}$ (e) $\frac{1024\pi}{3}$

18. $\int_0^{\pi^2/4} \frac{\cos \sqrt{x}}{\sqrt{x}} dx =$

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2 (e) $2\sqrt{2}$ (f) 3

19. Compute $\int (\sin^2 x + 8)^7 \sin x \cos x dx$.

(a) $\frac{1}{16}(\sin^2 x + 8)^8 + C$ (b) $(\cos^2 x + 8)^7 + C$ (c) $\frac{1}{2}(\sin^2 x + 8)^8 + C$
(d) $\frac{1}{8}(\sin^2(x + 8))^8 + C$ (e) $\frac{1}{8}(\cos^2 x + 8)^8 + C$ (f) $\frac{1}{32}(\sin^2 x + 8)^7 + C$

20. Suppose f is a continuous function and $\int_1^9 f(x) dx = 6$. Then $\int_1^3 xf(x^2) dx =$

(a) 6 (b) 4 (c) 3 (d) 2 (e) 1 (f) 0

21. $\int_0^1 \frac{e^{\arctan(x)}}{1+x^2} dx =$
(a) $e^{\pi/4} - 1$ (b) $e^{\pi/4}$ (c) $e - 1$ (d) $\frac{\pi}{4}$ (e) $\frac{\pi}{4} - e$ (f) $e^{\pi/2} - e^{\pi/4}$

22. Find the area between the graphs of $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \pi/4$
(a) $\frac{\sqrt{2}}{2}$ (b) $\sqrt{2}$ (c) $\sqrt{2} - 1$ (d) $\frac{\sqrt{2}}{2} + 1$ (e) $\frac{\sqrt{2}}{2} - 1$ (f) 1

23. $\int_0^{\pi/3} \sec^3 x \tan x \, dx$
(a) $\frac{\sqrt{2}}{6} - \frac{1}{6}$ (b) $\frac{\sqrt{3}}{6} - 1$ (c) $\frac{11}{3}$ (d) $\frac{7}{3}$ (e) $\frac{\sqrt{2}}{3} - \frac{1}{3}$ (f) $\frac{\sqrt{3}}{2} - \frac{1}{2}$

24. The base of a solid is the triangle in the xy -plane with vertices (0,0), (1,0) and (0,1). Cross-sections of the solid perpendicular to the x -axis are squares. What is the volume of the solid?
(a) $\frac{2}{3}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$ (e) $\frac{5}{4}$ (f) $\frac{1}{3}$

25. Find the volume of the solid obtained by rotating the area between the graphs of $y = x^2$ and $x = 2y$ around the y -axis.
(a) $\frac{2\pi}{45}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{96}$ (d) $\frac{\pi}{24}$ (e) $\frac{\pi}{180}$ (f) $\frac{3\pi}{64}$

26. A solid has as its base the region in the xy -plane the region between the curve $y = 1 - \frac{x^2}{4}$ and the x -axis. The cross-sections of the solid perpendicular to the x -axis are squares with one side in the base. What is the volume of the solid?
(a) $\frac{64}{15}$ (b) $\frac{16}{15}$ (c) $\frac{16}{5}$ (d) $\frac{128}{45}$ (e) $\frac{64}{45}$ (f) $\frac{32}{15}$

27. Find the average value of the function $f(x) = t\sqrt{16+t^2}$ on the interval $0 \leq x \leq 3$.
(a) $\sqrt{2}$ (b) $\frac{2\sqrt{2}}{9}$ (c) $\frac{32}{9}$ (d) $\frac{61}{3}$ (e) $\frac{\sqrt{2}}{3}$ (f) $\frac{61}{9}$

28. Find the volume obtained by rotating the region between the x -axis, the y -axis and the line $x + y = 1$ around the line $x = -2$.

- (a) $\frac{7\pi}{3}$ (b) $\frac{7\pi}{6}$ (c) $\frac{19\pi}{3}$ (d) $\frac{19\pi}{6}$ (e) $\frac{19\pi}{12}$ (f) $\frac{7\pi}{12}$
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29. Find the length of the part of the curve $y = \frac{1}{4}x^{3/2} - \frac{4}{3}x^{1/2}$ for $0 \leq x \leq 1$.

- (a) $\frac{7}{6}$ (b) $\frac{4}{3}$ (c) $\frac{17}{12}$ (d) $\frac{19}{12}$ (e) $\frac{23}{12}$ (f) $\frac{13}{6}$
-

30. Find the volume of the solid obtained by rotating the region between the graph of $y = \sin(x/2)$ and the x axis for $0 \leq x \leq 2\pi$ around the x -axis.

- (a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{4}$ (c) $\frac{\pi^2}{2}$ (d) π^2 (e) $2\pi^2$ (f) $4\pi^2$
-

31. Find the area between the graphs of $y = 1$ and $y = x^4$.

- (a) $\frac{8}{7}$ (b) $\frac{16}{9}$ (c) $\frac{16}{5}$ (d) $\frac{12}{7}$ (e) $\frac{8}{5}$ (f) 1
-

32. Find the volume of the solid obtained by rotating the area between the graphs of $y = x\sqrt{2-x}$ and $y = 0$ around the x -axis.

- (a) $\frac{\pi}{30}$ (b) $\frac{\pi}{24}$ (c) $\frac{4\pi}{3}$ (d) $\frac{\pi}{12}$ (e) $\frac{3\pi}{64}$ (f) $\frac{\pi}{180}$
-

33. $\int_0^1 \frac{x^3}{1+x^8} dx$

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{16}$ (e) $\frac{\pi}{20}$ (f) $\frac{\pi}{24}$
-

34. Find the volume obtained by rotating the square with corners at the points $(0, 0)$, $(0, 1)$, $(1, 1)$ and $(1, 0)$ around the line $x = 4$.

- (a) 3π (b) 5π (c) 7π (d) 9π (e) 11π (f) 13π
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35. Let $V(b)$ be the volume obtained by rotating the area between the x -axis and the graph of $y = \frac{1}{x^3}$ from $x = 1$ to $x = b$ around the x -axis. What is $\lim_{b \rightarrow \infty} V(b)$?

- (a) $\frac{\pi}{5}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$ (e) π (f) ∞
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36. Let $V(a)$ be the volume obtained by rotating the area between the x -axis and the graph of $y = \frac{1}{x^{3/2}}$ from $x = a$ to $x = 1$ around the y -axis. What is $\lim_{a \rightarrow 0+} V(a)$?

- (a) $\frac{\pi}{5}$ (b) $\frac{4\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) 4π (e) $\frac{6\pi}{5}$ (f) ∞
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37. Find the surface area obtained by rotating the part of the curve $y = 4 - x^2$ for $0 \leq x \leq 2$ around the y -axis.

- (a) $\frac{\pi}{6}(5\sqrt{5} - 1)$ (b) $\frac{\pi}{6}(17\sqrt{17} - 1)$ (c) $\frac{\pi}{6}(37\sqrt{37} - 1)$
 (d) $\frac{\pi}{6}(65\sqrt{65} - 1)$ (e) $\frac{\pi}{6}(101\sqrt{101} - 1)$ (f) $\frac{\pi}{6}(145\sqrt{145} - 1)$
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38. $\int_0^{\sqrt{\pi}} x \sin^2(x^2) dx$

- (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$ (e) $\frac{\pi}{4}$ (f) $\frac{3\pi}{4}$
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39. Find the area between the graphs of $y = x^4 + 4x^2$ and $y = 4x^3$.

- (a) $\frac{1}{30}$ (b) $\frac{8}{15}$ (c) $\frac{16}{15}$ (d) $\frac{27}{10}$ (e) $\frac{81}{10}$ (f) $\frac{512}{15}$
-

40. The average value of the function $f(x)$ on the interval $[0, b]$ is $\sqrt{b}$. What is $f(x)$?

- (a) $(1+x)e^x$ (b) $\frac{e^x}{1+x}$ (c) $\cos x + x \sin x$ (d) $\sin x + x \cos x$ (e) $\frac{3}{2\sqrt{x}}$ (f) $\frac{3\sqrt{x}}{2}$
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41. Calculate the length:

- (a) of the part of $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 1$. (Answer: $4/3$)
 (b) of the part of $y = x^{2/3}$ from $x = -1$ to $x = 8$ (careful!) (Answer: $\frac{80\sqrt{10} + 13\sqrt{13} - 16}{27}$)
 (c) of the part of $y = \ln(\cos x)$ for $0 \leq x \leq \pi/4$. (Answer: $\ln(\sqrt{2} + 1)$)
 (d) of the part of $y = \ln x$ for $1 \leq x \leq 2$
 (Answer: $\sqrt{5} - \sqrt{2} + \frac{1}{2} \ln(\sqrt{5} - 1) - \frac{1}{2} \ln(\sqrt{2} - 1) - \frac{1}{2} \ln(\sqrt{5} + 1) + \frac{1}{2} \ln(\sqrt{2} + 1)$)
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42. Calculate the surface area obtained by rotating:

(a) $y = \sqrt{x}$ around the x axis for $0 \leq x \leq 4$ (Answer: $\frac{\pi}{6}(17\sqrt{17} - 1)$)

(b) $y = x^3$ around the x axis for $1 \leq x \leq 2$ (Answer: $\frac{\pi}{27}(145\sqrt{145} - 10\sqrt{10})$)

(c) $y = \sqrt{9 - x^2}$ around the y axis for $1 \leq x \leq 3$. (Answer: $12\pi\sqrt{2}$)

(d) $y = x^2$ around the x axis for $0 \leq x \leq 1$. (Answer: $\frac{\pi}{32}(18\sqrt{5} - \ln(\sqrt{5} + 2))$)