

1. Evaluate

$$\int_0^1 15x^3 \sqrt{1-x^2} dx$$

- (A) $\sqrt{3}$ (C) $\frac{\pi}{3}$ (E) $\sqrt{2}$ (G) The integral diverges
(B) $\frac{1}{2}$ (D) 2 (F) 1 (H) None of these

2. Evaluate

$$\int_3^4 \frac{16x}{x^4 - 16} dx$$

- (A) 0 (C) $\frac{13}{5}$ (E) $\ln\left(\frac{13}{5}\right)$ (G) $\ln\left(\frac{39}{25}\right)$
(B) $\ln\left(\frac{26}{25}\right)$ (D) $\ln\left(\frac{52}{25}\right)$ (F) $\ln\left(\frac{19}{5}\right)$ (H) None of these

3. Evaluate

$$\int_{12}^{\infty} \frac{1}{\sqrt{x}(x+4)} dx$$

- (A) $\frac{\pi}{2}$ (C) $\frac{\pi}{6}$ (E) $\frac{\pi}{3}$ (G) 16π
(B) 4π (D) $\frac{4\pi}{3}$ (F) 12π (H) None of these

4. The function below is a probability density function. Find the value of C .

$$f(x) = \begin{cases} \frac{8C}{x^2 - 4} & \text{if } x \geq 4 \\ 0 & \text{otherwise} \end{cases}$$

- (A) 2 (C) 1 (E) $\frac{1}{\ln 3}$ (G) $\frac{1}{3}$
(B) $\frac{1}{\ln 9}$ (D) $\frac{1}{\ln 6}$ (F) $\frac{1}{\ln 27}$ (H) None of these

5. The Laplace transform $L\{f(x)\}$ of a function $f(x)$ is defined as

$$L\{f(x)\} = \int_0^{\infty} e^{-sx} f(x) dx \quad \text{for } x \geq 0 \text{ with } s \text{ being a complex number}$$

Find the Laplace transform of $f(x) = x$.

- (A) $\frac{1}{s^2}$ (C) $\frac{2}{s^3}$ (E) $\frac{3}{s^4}$ (G) s
(B) $\frac{1}{s+1}$ (D) $\frac{1}{2}$ (F) 1 (H) None of these

6. Solve the differential equation

$$\frac{dy}{dx} + \frac{e^x y}{1+e^x} = \frac{\sin x}{1+e^x} \quad \text{with } y(0) = \frac{1}{2}$$

Find $y(\pi)$.

- (A) $\frac{2}{1+e^\pi}$ (C) $\frac{3}{1+e^\pi}$ (E) $\frac{4}{1+e^\pi}$ (G) $\frac{6}{1+e^\pi}$
(B) $\frac{1}{1+e^\pi}$ (D) $\frac{8}{1+e^\pi}$ (F) $\frac{12}{1+e^\pi}$ (H) None of these

7. Solve the differential equation

$$x \frac{dy}{dx} = y \ln x \quad \text{with } y(1) = 4$$

Find $y(e)$.

- (A) 0 (C) 1 (E) e^2 (G) e
(B) $2\sqrt{e}$ (D) $3\sqrt{e}$ (F) $4\sqrt{e}$ (H) None of these

ANSWERS:

1. D
2. G
3. C
4. B
5. A
6. C
7. F