MATH 104 – Practice Midterm Exam 1

There will be eight problems on the midterm, chosen more or less from the following types. The midterm problems have Multiple Choice answers but we will be grading the work.

1. (a) What are the domain and range of the function $f(x) = \log_2(16 - x^2)$?

   (b) On the part of the domain of f where $x \geq 0$, f is an invertible function. Calculate the inverse function f^{-1}. What are its domain and range?

2. Calculate $\displaystyle\int \frac{x^2\,dx}{x^3+1}$.

3. The half-life of Carbon-14 is approximately 5600 years. How many years does it take a 152 mg sample of Carbon-14 to decrease to 19 mg ?

4. Find the area of the region enclosed between the graph of $y = x\sin x^2$, the line $y = \sqrt{\frac{\pi}{2}}$ and the y-axis for $0 \leq x \leq \sqrt{\frac{\pi}{2}}$.

5. Calculate the volume of the solid whose base is the unit disk $x^2 + y^2 = 1$ and whose cross sections perpendicular to the x axis are isosceles right triangles (i.e., 45-45-90 triangles) with their hypotenuses in the base.

6. Calculate the volume of the solid formed by revolving the region between the graphs of $y = x\sqrt{1+x^3}$, $y = 0$, $x = 0$ and $x = 2$ around the y-axis.

7. Calculate the volume of the solid formed by revolving the region between the graph of $y = \dfrac{x}{\sqrt{1+x^3}}$ and the x-axis for $0 \leq x \leq 2$ around the x-axis.

8. Calculate the length of the part of the curve $y = \frac{1}{3}(x^2+2)^{3/2}$ between $x = 0$ and $x = 2$.

9. Calculate the area of the surface you get by revolving the part of the curve $y = 1 - x^2$ for $0 \leq x \leq 1$ around the y-axis.

10. Find the area in the first quadrant between the graph of $y = e^x$ and the tangent line to the graph at the point $(1, e)$.

11. Suppose one end of a straight line segment of length 1 is fixed at the origin. Among all such segments, which one gives the maximum volume of a solid of revolution when it is revolved around the x-axis? What is the maximum volume?

12. (a) What are the domain and range of the function $f(x) = e^x + \ln x$?

    (b) Explain how you know that this function has an inverse. What are the domain and range of f^{-1}?

    (c) Calculate the value of $\dfrac{df^{-1}}{dx}$ when $x = e$.

13. Let $y = \sqrt{\dfrac{(4x+3)^8(x+4)}{\sqrt[3]{x^2+9}}}$. Calculate $\dfrac{dy}{dx}$.

14. Calculate the area of the region bounded by the x-axis, the curve $y = 3^{1-x}$ and the lines $x = -1$ and $x = 1$. What volume do you get if you revolve this region around the x-axis?

15. Assume $a > 0$. Evaluate $\displaystyle\int \frac{\log_a x}{x}\,dx$.

16. Calculate the volume of the torus formed by revolving the disc $x^2 + y^2 \leq 4$ about the line $x = 9$.