**Solution outlines for Palvannan midterm 3**

**1**. Starting from $a_2$, all the $a_n$ are positive. If there is a limit $L$, then it must satisfy

$$L = \sqrt{6+L} \quad \text{i.e.,} \quad L^2 = 6 + L$$

which is the same as $(L-3)(L+2) = 0$. But $L$ cannot be $-2$ since all the $a_n$ are positive. Therefore $L = 3$.

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**2**. Using the ratio test, we get

$$\lim_{n\to\infty} \frac{n+1}{(2n-1)(2n-2)} = \frac{1}{4} < 1$$

therefore the series converges absolutely.

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**3**. The terms decrease to zero, so by the alternating series test, the series converges. But $\ln n < n^{0.1}$ for large $n$ so

$$\sum \frac{1}{(\ln n)^2} > \sum \frac{1}{n^{0,2}}$$

which diverges, so the series only converges conditionally.

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**4**. The terms of both series are positive, so they either converge absolutely or else they diverge.

(a) We have

$$\frac{\pi^n}{4^n + n^5} < \frac{\pi^n}{4^n} = \left(\frac{\pi}{4}\right)^n$$

Since $\pi < 4$, we have directly compared our series to a convergent geometric series. Therefore by the direct comparison test, the original series converges (absolutely).

(b) Use the ratio test:

$$\lim_{n\to\infty} \frac{(n+1)^{100}}{2^{n+1}} \cdot \frac{2^n}{n+100} = \frac{1}{2} < 1$$

so the series is absolutely convergent.

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**5**. (a) If $\sum a_n = \frac{1}{2}$ then we must have $a_n \to 0$, therefore $(\frac{1}{2})^{a_n} \to 1$. But then $\sum(\frac{1}{2})^{a_n}$ diverges by the $n$th term test.

(b) Use the ratio test:

$$\lim_{n\to\infty} \frac{(\frac{1}{2})^{n+1+a_{n+1}}}{(\frac{1}{2})^{n+a_n}} = \lim_{n\to\infty} \frac{1}{2}\frac{(\frac{1}{2})^{a_{n+1}}}{(\frac{1}{2})^{a_n}} = \frac{1}{2}\cdot\frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{2} < 1$$

so the series $\sum(\frac{1}{2})^{n+a_n}$ converges absolutely.