Solution outlines for Palvannan midterm 2

1. Complete the square:

$$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{1-1+2x-x^2}} = \int \frac{dx}{\sqrt{1-(x-1)^2}}$$

Now let $x-1=\sin\theta$:

$$\cdots = \int \frac{\cos\theta\,d\theta}{\cos\theta} = \theta + C = \arcsin(x-1)+C.$$

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2. (a) The result of the long division is $x^3+8=(x+2)(x^2-2x+4)$.
(b) Using (a),

$$\begin{aligned}\int_1^2 \frac{x}{x^3+8}\,dx &= \int_1^2 \frac{x}{(x+2)(x^2-2x+4)}\,dx = \int_1^2 \frac{-\frac{1}{6}}{x+2} + \frac{\frac{1}{6}(x+2)}{x^2-2x+4}\,dx \\ &= \frac{1}{6}\int_1^2 \frac{-1}{x+2} + \frac{x-1}{(x-1)^2+3} + \frac{3}{(x-1)^2+3}\,dx \\ &= \frac{1}{6}\left(-\ln(x+2) + \frac{1}{2}\ln((x-1)^2+3) + \sqrt{3}\arctan\frac{x-1}{\sqrt{3}}\right)\Bigg|_1^2 \\ &= \frac{1}{6}\left(\ln\frac{3}{4} + \frac{1}{2}\ln\frac{4}{3} + \sqrt{3}\arctan\frac{1}{\sqrt{3}}\right) \\ &= \frac{1}{6}\left(\frac{1}{2}\ln\frac{3}{4} + \sqrt{3}\frac{\pi}{6}\right)\end{aligned}$$

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3. (a) is the probability that the tire will fail between 10,000 and 20,000 miles (iii).
(b) is the probability that the tire will fail before 20,000 miles (i)
(c) is the probability that the tire will fail after 10,000 miles (iii)
(d) is the probability that the tire will fail before 15,000 miles, since it can't fail before 0 miles (iii).

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4. Using the trig identity $\sin 2x = 2\sin x\cos x$, rewrite the integral as

$$I = \frac{1}{2}\int e^x \sin 2x\,dx.$$

Then integrate by parts with $u = \sin 2x$ and $dv = e^x\,dx$ and get

$$I = \frac{1}{2}\left(e^x \sin 2x - 2\int e^x \cos 2x\,dx\right)$$

Now integrate by parts again with $u = \cos 2x$ and $dv = e^x\,dx$ and get

$$I = \frac{1}{2}\left(e^x \sin 2x - 2\left(e^x \cos 2x + \int 2e^x \sin 2x\,dx\right)\right) = \frac{1}{2}e^x \sin 2x - e^x \cos 2x - 2\int e^x \sin 2x\,dx$$

The last term is $-4I$; add it back to the left to get

$$5I = \frac{1}{2}e^x \sin 2x - e^x \cos 2x$$

Therefore

$$I = \int e^x \sin x \cos x\,dx = \frac{1}{10}e^x(\sin 2x - 2\cos 2x) + C.$$

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5. Because we know that $-1 < \cos e^x < 1$, we know that $\dfrac{5+\cos e^x}{x^2+1}$ is positive and that it is less than $\dfrac{6}{x^2+1}$. Therefore

$$\int_0^\infty \frac{5+\cos e^x}{x^2+1} < 6\int_0^\infty \frac{1}{x^2+1}\,dx = \frac{\pi}{3}.$$

Therefore the original integral converges by the direct comparison test.