**Solution outlines for Palvannan midterm 1**

**1**. Take the derivative:

$$y' = x^{3/2} - \frac{x^{-3/2}}{4}$$

and see that *it's one of those*. Therefore

$$L = \frac{2x^{5/2}}{5} - \frac{x^{-1/2}}{2}\Bigg|_1^4 = \frac{64}{5} - \frac{1}{4} - \left(\frac{2}{5} - \frac{1}{2}\right) = \frac{253}{20}.$$

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**2**. (a) Since the region is revolved around the $x$-axis, to get disks we take vertical slices. The disks have radius $\arccos x$ and thickness $dx$, so the volume is

$$V = \int \pi r^2 t = \pi \int_0^1 (\arccos x)^2 \, dx.$$

(b) To use shells, we need horizontal slices. The shells will have radius $y$, height $\cos y$ and thickness $dy$. So this way the volume is

$$V = \int 2\pi r h t = 2\pi \int_0^{\pi/2} y \cos y \, dy.$$

(Either way, the integral evaluates to $\pi^2 - 2\pi$.

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**3**. Using horizontal sections — rotate a horizontal section around the line $y = 5$ and get a shell with radius $5 - y$, height $\dfrac{1}{y^3} - \dfrac{1}{8}$ and thickness $dy$. So the volume is

$$\begin{aligned} V &= 2\pi \int_1^2 (5-y)\left(\frac{1}{y^3} - \frac{1}{8}\right) dy = 2\pi \int_1^2 \frac{5}{y^3} - \frac{5}{8} - \frac{1}{y^2} + \frac{y}{8} \, dy \\ &= 2\pi \left(-\frac{5}{2y^2} - \frac{5y}{8} + \frac{1}{y} + \frac{y^2}{16}\right)\Bigg|_1^2 = \frac{15\pi}{8}. \end{aligned}$$

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**4**. Since the density is constant, we can ignore it for the purposes of computing the center of mass. First we need the area of the region. Using horizontal slices,

$$A = \int_0^2 y - (y^2 - y)\, dy = \int_0^2 2y - y^2\, dy = y^2 - \frac{y^3}{3}\Big|_0^2 = 4 - \frac{8}{3} = \frac{4}{3}.$$

Then

$$\overline{y} = \frac{1}{A}\int_0^2 2y^2 - y^3\, dy = \frac{3}{4}\left(\frac{2y^3}{3} - \frac{y^4}{4}\right)\Big|_0^2 = \frac{3}{4}\left(\frac{16}{3} - 4\right) = 4 - 3 = 1.$$

Next,

$$\begin{aligned}
\overline{x} &= \frac{1}{A}\int_0^2 \frac{1}{2}\left(y^2 - (y^2 - y)^2\right)\, dy = \frac{1}{A}\int_0^2 \frac{1}{2}(y^2 - (y^4 - 2y^3 + y^2))\, dy \\
&= \frac{3}{4}\int_0^2 2y^3 - y^4\, dy = \frac{3}{8}\left(\frac{y^4}{2} - \frac{y^5}{5}\right)\Big|_0^2 = \frac{3}{8}\left(8 - \frac{32}{5}\right) \\
&= 3 - \frac{12}{5} = \frac{3}{5}
\end{aligned}$$

So the center of mass is the point $\left(\frac{3}{5}, 1\right)$.

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**5**. (i) True, because $x^5$ is an odd function and $\cos x$ is even, so their product is odd and so the integral of the product over an interval symmetric around 0 is zero.

(ii) False. The integral is zero, because $x^4$ is even and $\sin x$ is odd, so the product is odd.

(iii) This is true.

(iv) Technically false, since it should say that the line does not cut through the *region's* interior (rather than the plane's interior), but otherwise true.

(v) False – this i.e. the $y$ coordinate of the centroid. The $x$ coordinate of the centroid is zero.