Midterm Exam #3 for Math 104-004, Fall 2016

Name : ________________________________ Recitation day and hours: ________________

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: ________________________________ Date: __________________________

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- Box your final answers.
- You have 50 minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate. Answers with little or no justification will get no credit.
- You may use both sides of one 8.5 by 11 inch sheet of notes.
- **NO** books, laptops, cell phones, calculators, or any other electronic devices may be used during the exam.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

1. (10 points) Consider a sequence $\{a_n\}_{n=1}^{\infty}$ defined using the following recursion formula:

$$a_1 = -1, \qquad a_{n+1} = \sqrt{6 + a_n}, \text{ for all } n \geq 1.$$

Assume that the limit $\lim_{n\to\infty} a_n$ exists and equals L. Find L. Justify your answer completely.

2. (10 points) Does the series $\sum_{n=1}^{\infty}(-1)^n\frac{n!}{(2n-1)!}$ converge absolutely, converge conditionally or diverge? Justify your answer completely.

3. (10 points) Does the series $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{(\ln(n))^2}$ converge absolutely, converge conditionally or diverge? Justify your answer completely.

4. (10 points) For each $n \geq 1$, let

$$a_n = \frac{\pi^n}{4^n + n^5}, \qquad b_n = \frac{n^{100}}{2^n}.$$

(a) (5 points) Does the series $\sum_{n=1}^{\infty} a_n$ converge absolutely, converge conditionally or diverge? Justify your answer completely.

(b) (5 points) Does the series $\sum_{n=1}^{\infty} b_n$ converge absolutely, converge conditionally or diverge? Justify your answer completely.

5. Suppose the series $\sum_{n=1}^{\infty} a_n$ converges to $\frac{1}{2}$.

(a) (5 points) Does the series $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{a_n}$ converge absolutely, converge conditionally or diverge? Justify your answer completely.

(b) (5 points) Does the series $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n+a_n}$ converge absolutely, converge conditionally or diverge? Justify your answer completely.

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