MATH 104 – Practice Final Exam

The exam will look something like this (in terms of length, type of problems etc):

1. A scientist collects data that relate two variables, x and y. Instead of plotting y as a function of x, she plots $\log_{10} y$ as a function of x, and gets a line whose slope is -2 and whose intercept on the vertical axis is 8. What equation describes y as a function of x?

2. Find the arc length of the portion of the graph of the function $f(x) = 2 \sec x$ that lies above the interval $[0, \frac{\pi}{4}]$.

3. Find the volume of the solid of revolution obtained by revolving the area between the graphs of $y = e^{-x}$ and $y = -e^{-x}$ for x between 0 and $\ln 10$ around the y axis.

4. Solve the initial-value problem $x^5 y' - (\ln x) y = \ln x$, $y(1) = 0$.

5. Psychologists have ascertained that the rate at which a person can memorize a set of unrelated facts is proportional to the number of unlearned facts. If M is the (large) total number of facts to be learned, and a person learns 10 of them in the first hour, how long will it take until the person knows half of the facts?

6. Calculate $\lim\limits_{x \to \infty} \left(\dfrac{2x - 1}{2x + 1} \right)^{2x+3}$.

7. Evaluate $\displaystyle\int_1^\infty \frac{\arctan x}{x^2}\, dx$, or else show that the integral diverges.

8. Calculate $\displaystyle\int \frac{e^x dx}{e^{2x} - 5e^x + 4}$.

9. Calculate $\displaystyle\int_0^{\pi/4} \cos^5 x \sin 2x \, dx$.

10. Sketch the graph of $y = \dfrac{\ln x}{x}$ for $x \in (0, \infty)$. Indicate all "interesting" points (max/min, inflection etc) and asymptotes.

11. Find the area of the region bounded by the (upper part of) the hyperbola $y^2 - x^2 = 1$ and the line $y = 4$.

12. Find the value of C for which the integral

$$\int_0^\infty \left(\frac{1}{\sqrt{x^2 + 9}} - \frac{C}{x + 4} \right) dx$$

converges. Evaluate the integral for this value of C.

13. Find the limit of the sequence $\{\sqrt{n^2+2n}-n\}$ or else explain why it does not converge.

14. Investigate the convergence (absolute/conditional/divergence) of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

15. Use an appropriate series approximation to calculate

$$\int_0^{0.1} \frac{dx}{\sqrt{1+x^4}}$$

to within 10^{-4}. Be sure to explain how you know your error is this small.

16. For (precisely) which x does the power series

$$\sum_{n=0}^{\infty} \frac{4^n(x-5)^n}{\sqrt{n+1}}$$

converge? (Be sure to check the endpoints of your interval!)

17. (a) Write the third-degree Taylor polynomial for the function $y = x^{1/4}$ expanded around $a = 16$.

    (b) If the value of this polynomial at $x = 15$ is taken as an approximation for $(15)^{1/4}$, give an estimate of the error.

18. Find the first four non-zero terms in the power series of the solution of the initial-value problem $y' + y = x$, $y(0) = 0$.

19. Use the idea behind the proof of the integral test to show that

$$\int_1^n \ln x\, dx < \ln(n!) < \int_2^{n+1} \ln x\, dx.$$

From this, conclude that

$$\frac{n^n}{e^n}e < n! < \frac{(n+1)^{n+1}}{e^n}\frac{e}{4}.$$