MATH 104 – Practice Problems for Exam 3

There are too many problems here for one exam, but they're good practice!

For each of the following series, say whether it converges or diverges, and explain why.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{4^n+n^4}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

4. $\displaystyle\sum_{n=1}^{\infty} n \sin\frac{1}{n}$

5. How many terms of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ are needed to get a partial sum that is within 0.001 of the actual sum of the series? Give the smallest possible number of terms.

6. Does $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n}$ converge? If so, what is the sum?

7. For what values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^p}{2+n^3}$ converge?

8. Find the precise interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(2x-5)^n}{n^2 4^n}.$$

9. Which of the following is the beginning of the Maclaurin series for $\arctan(x^2)$?

(A) $x^2 - \dfrac{x^4}{3} + \dfrac{x^6}{5} - \dfrac{x^8}{7} + \ldots$

(B) $\dfrac{x^2}{3} - \dfrac{x^6}{6} + \dfrac{x^{10}}{9} - \dfrac{x^{14}}{12} + \ldots$

(C) $x^2 - 2x^4 + 3x^6 - 4x^8 + \ldots$

(D) $1 + x^4 + 3x^8 + 4x^{12} + \ldots$

(E) $\dfrac{x^2}{3} + \dfrac{x^6}{6} + \dfrac{x^{10}}{9} + \dfrac{x^{14}}{12} + \ldots$

(F) $x^2 - \dfrac{x^6}{3} + \dfrac{x^{10}}{5} - \dfrac{x^{14}}{7} + \ldots$

10. Let $F(x) = \int_0^x \cos\sqrt{t}\,dt$. Which of the following is the beginning of the Maclaurin series for F?

(A) $1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720} + \dots$

(B) $x - \frac{x^2}{4} + \frac{x^3}{72} - \frac{x^4}{2880} + \dots$

(C) $x - \frac{x^2}{2} + \frac{x^4}{6} - \frac{x^6}{24} + \dots$

(D) $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$

(E) $x - \frac{x^2}{3} + \frac{x^4}{15} - \frac{x^5}{105} + \dots$

(F) $x + \frac{x^2}{3} + \frac{x^4}{15} + \frac{x^5}{105} + \dots$

11. Use the first two non-zero terms of an appropriate series to give an approximation of

$$\int_0^1 \sin(x^2)\,dx.$$

Give (with explanation) an estimate of the error (difference between your approximation and the actual value of the integral).

12. We would like to estimate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^4+3}$, by using the sum of the first ten terms. Of course, the exact error is the sum of all the terms from the 11th on, i.e., $\sum_{n=11}^{\infty} \frac{1}{n^4+3}$. Show that this error is less than $1/3000$ by comparing this with the sum of $1/n^4$ and then by estimating this latter sum using an appropriate integral.

13. Find the center and radius of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n\sqrt{1+n}}{n^2}(x-2)^n.$$

(In other words, find the largest *open* interval on which the series converges).

14. Write the second-degree Taylor polynomial for $f(x) = \sqrt{x}$ centered at $a = 25$. Use this polynomial from to estimate $\sqrt{26}$. Also, give an estimate of the error.

15. Determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}$ converges or diverges. Please explain carefully.

16. Does the sequence $\{(n^2+n)^{1/n}\}$ converge or diverge? If it converges, calculate its limit.

17. Let $a_n = \displaystyle\int_n^\infty \frac{1}{1+(x^2+x^4)^2}dx$. Does the sequence $\{a_n\}$ converge? If so, what is the limit?

18. Determine whether the following series converges:

$$\sum_{n=3}^{\infty} \ln(1+\frac{1}{n^2})$$

19. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$ converge or diverge? Be sure to justify your answer.

20. Find the limit.

$$\lim_{x\to 0} \frac{\cos x - 1}{e^{x^2}-1}$$

The rest of these are problems from exams the last few times I've given the course.

21. What is the limit of the sequence $\left\{\arctan\left(\dfrac{n^2}{1+n^2}\right)\right\}$?

(a) 0 (b) 1 (c) $\dfrac{\pi}{4}$ (d) $\ln 3$ (e) $\ln\dfrac{1}{9}$ (f) diverges

22. Does the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-\sqrt{n}}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

23. What is the limit of the sequence $\left\{\cos\left(\dfrac{\sqrt{n}}{1+n}\right)\right\}$?

(a) 0 (b) 1 (c) $\dfrac{\pi}{4}$ (d) $\ln 3$ (e) $\ln\dfrac{1}{9}$ (f) diverges

24. Does the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

25. Does the series $\sum_{n=1}^{\infty} \frac{n!}{n^5}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

26. What is the limit of the sequence $\left\{n^2\left(1-\cos\frac{1}{n}\right)\right\}$?

(a) 1 (b) -1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$ (e) $-\frac{\sqrt{3}}{2}$ (f) diverges

27. Does the series $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

28. Does the series $\sum_{n=1}^{\infty} \frac{1}{n^n}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

29. Does the series $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

30. Does the series $\sum_{n=0}^{\infty} \frac{2n+3}{(n^2+3n+6)^2}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

31. For which values of x does the series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{5^n}$ converge? What is the sum?

(a) $-1 < x < 1$, $\frac{5}{x-4}$ (b) $-1 < x < 1$, $\frac{5}{9-x}$ (c) $-1 < x < 9$, $\frac{5}{x-4}$

(d) $-1 < x < 9$, $\frac{5}{9-x}$ (e) $-1 < x < 1$, $\frac{4}{5-x}$ (f) $-1 < x < 9$, $\frac{4}{5-x}$

32. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{3^n+4^n}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

33. Does the series $\sum_{n=1}^{\infty} \frac{n^5}{n!}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

34. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+n^3)}{n^4+1}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

35. Does the series $\sum_{n=1}^{\infty} \frac{(2n)!}{2^n(n!)^2}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

36. Does the series $\sum_{n=1}^{\infty} \frac{2^n(n!)^2}{(2n)!}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

37. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \ln n}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

38. For which values of p does the series $\sum_{n=0}^{\infty} \frac{1}{(n^2+1)^p}$ converge?

(a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p < \ln 5 - \ln 3$ (e) $p < \frac{\ln 5}{\ln 3}$ (f) $p < 0$

39. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n3^n}$ converge?

(a) $-1 < x \leq 5$ (b) $-1 < x < 5$ (c) $-1 \leq x \leq 5$

(d) $5/3 \leq x \leq 7/3$ (e) $-3 < x < 3$ (f) $-3 \leq x \leq 3$

40. For which values of p does the series $\sum_{n=0}^{\infty} \ln(1+n^p)$ converge?

(a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p < \ln 5 - \ln 3$ (e) $p < \dfrac{\ln 5}{\ln 3}$ (f) $p < 0$

41. For which values of x does the series $\sum_{n=1}^{\infty} \dfrac{(x-2)^n}{n^2 3^n}$ converge?

(a) $-1 < x \leq 5$ (b) $-1 < x < 5$ (c) $-1 \leq x \leq 5$
(d) $5/3 \leq x \leq 7/3$ (e) $-3 < x < 3$ (f) $-3 \leq x \leq 3$

42. For which values of p does the series $\sum_{n=0}^{\infty} \dfrac{3^{np}}{4^n + 5^n}$ converge?

(a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p < \ln 5 - \ln 3$ (e) $p < \dfrac{\ln 5}{\ln 3}$ (f) $p < 0$

43. For which values of x does the series $\sum_{n=1}^{\infty} \dfrac{3^n (x-2)^n}{n^2}$ converge?

(a) $-1 < x \leq 5$ (b) $-1 < x < 5$ (c) $-1 \leq x \leq 5$
(d) $5/3 \leq x \leq 7/3$ (e) $-3 < x < 3$ (f) $-3 \leq x \leq 3$

44. What are the first few nonzero terms of the Maclaurin series for $f(x) = xe^{3x}$?

(a) $1 - 2x^2 + \dfrac{2}{3}x^4 - \dfrac{4}{45}x^6 + \cdots$ (b) $x + \dfrac{1}{4}x^2 + \dfrac{1}{8}x^3 + \dfrac{1}{12}x^4 + \cdots$
(c) $x^2 + \dfrac{1}{4}x^3 + \dfrac{1}{16}x^4 + \dfrac{1}{64}x^5 + \cdots$ (d) $1 + 2x^2 + \dfrac{4}{3}x^4 + \dfrac{4}{45}x^6 + \cdots$
(e) $x^2 - \dfrac{1}{4}x^3 + \dfrac{1}{16}x^4 - \dfrac{1}{64}x^5 + \cdots$ (f) $x + 3x^2 + \dfrac{9}{2}x^3 + \dfrac{9}{2}x^4 + \cdots$

45. Which of the following gives the value of $\int_0^{1/2} e^{-x^5}\,dx$ correct to within 0.0001 (i.e., within 1/10000)?

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{2} - \dfrac{1}{120}$ (c) $\dfrac{1}{24} - \dfrac{1}{2688}$ (d) $\dfrac{1}{2} - \dfrac{1}{320}$ (e) $\dfrac{1}{2} - \dfrac{1}{384}$ (f) $\dfrac{1}{24} - \dfrac{1}{1024}$

46. What are the first few nonzero terms of the Maclaurin series for $f(x) = \dfrac{4x^2}{4-x}$?

(a) $1 - 2x^2 + \dfrac{2}{3}x^4 - \dfrac{4}{45}x^6 + \cdots$ (b) $x + \dfrac{1}{4}x^2 + \dfrac{1}{8}x^3 + \dfrac{1}{12}x^4 + \cdots$

(c) $x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^4 + \frac{1}{64}x^5 + \cdots$

(d) $1 + 2x^2 + \frac{4}{3}x^4 + \frac{4}{45}x^6 + \cdots$

(e) $x^2 - \frac{1}{4}x^3 + \frac{1}{16}x^4 - \frac{1}{64}x^5 + \cdots$

(f) $x + 3x^2 + \frac{9}{2}x^3 + \frac{9}{2}x^4 + \cdots$

47. Which of the following gives the value of $\int_0^{1/2} \cos(x^2)\,dx$ correct to within 0.0001 (i.e., within 1/10000)?

(a) $\frac{1}{2}$ (b) $\frac{1}{2} - \frac{1}{120}$ (c) $\frac{1}{24} - \frac{1}{2688}$ (d) $\frac{1}{2} - \frac{1}{320}$ (e) $\frac{1}{2} - \frac{1}{384}$ (f) $\frac{1}{24} - \frac{1}{1024}$

48. What are the first few nonzero terms of the Maclaurin series for $f(x) = \cos(2x)$?

(a) $1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \cdots$

(b) $x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{12}x^4 + \cdots$

(c) $x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^4 + \frac{1}{64}x^5 + \cdots$

(d) $1 + 2x^2 + \frac{4}{3}x^4 + \frac{4}{45}x^6 + \cdots$

(e) $x^2 - \frac{1}{4}x^3 + \frac{1}{16}x^4 - \frac{1}{64}x^5 + \cdots$

(f) $x + 3x^2 + \frac{9}{2}x^3 + \frac{9}{2}x^4 + \cdots$

49. Which of the following gives the value of $\int_0^{1/2} \arctan(x^2)\,dx$ correct to within 0.0001 (i.e., within 1/10000)?

(a) $\frac{1}{2}$ (b) $\frac{1}{2} - \frac{1}{120}$ (c) $\frac{1}{24} - \frac{1}{2688}$ (d) $\frac{1}{2} - \frac{1}{320}$ (e) $\frac{1}{2} - \frac{1}{384}$ (f) $\frac{1}{24} - \frac{1}{1024}$

MATH 104-004 – Third Midterm Exam - Fall 2011

1. What is the limit of the sequence $\{\sin(\arctan(\ln(n)))\}$?

1. What is the limit of the sequence $\{\cos(\arctan(\ln(n)))\}$?

1. What is the limit of the sequence $\{\ln(\cos(\arctan(n)))\}$?

(a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\ln 2$ (e) $\frac{\pi}{2}$ (f) diverges

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2. Does the series $\sum_{n=2}^{\infty} \frac{n}{n^3 - \sqrt{n} + 1}$ converge or diverge? (Why?)

2. Does the series $\sum_{n=2}^{\infty} \frac{n^2}{n^3 - \sqrt{n} + 1}$ converge or diverge? (Why?)

2. Does the series $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n^2 - n + 1}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

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3. Does the series $\sum_{n=1}^{\infty} \frac{2^n}{n^5}$ converge or diverge? (Why?)

3. Does the series $\sum_{n=1}^{\infty} \frac{n^5}{2^n}$ converge or diverge? (Why?)

3. Does the series $\sum_{n=1}^{\infty} \frac{n^5 5^n}{n!}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

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4. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2 + n)}{n^3 + 2n + 1}$ converge absolutely, converge conditionally, or diverge? (Why?)

4. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2 + n^3)}{n^5 + 1}$ converge absolutely, converge conditionally, or diverge? (Why?)

4. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2 + n^3)}{n^4 + \ln n}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

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5. If it converges, find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n2^n}$ If the series diverges, explain why.

5. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{9^n (2n)!}$ If the series diverges, explain why.

5. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!\, 2^n}$ If the series diverges, explain why.

(a) $\ln 2$ (b) $\ln 3 - \ln 2$ (c) $1/\sqrt{e}$ (d) $1/2$ (e) $1/e^2$ (f) Diverges

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6. Does the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n!)}$ converge absolutely, converge conditionally, or diverge? (Why?)

6. Does the series $\sum_{n=2}^{\infty} \frac{(-1)^n \sqrt{n^3+1}}{\ln(n!)}$ converge absolutely, converge conditionally, or diverge? (Why?)

6. Does the series $\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n!)}{\sqrt{n^5+1}}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

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7. For which values of p does the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{1+n^p}}$ converge?

7. For which values of p does the series $\sum_{n=1}^{\infty} \sqrt{1+n^p}$ converge?

7. For which values of p does the series $\sum_{n=1}^{\infty} \frac{n}{1+n^p}$ converge?

(a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p > 2$ (e) $p < 0$ (f) no value of p

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8. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2}$ converge?

8. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{1+2^n}$ converge?

8. For which values of x does the series $\sum_{n=1}^{\infty} \frac{nx^n}{2^n}$ converge?

(a) $-1 < x < 3$ (b) $0 < x < 2$ (c) $-1 \le x \le 3$

(d) $0 \le x \le 2$ (e) $-2 < x < 2$ (f) $-2 \le x \le 2$

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9. What are the first few nonzero terms of the Maclaurin series for $f(x) = x\cos(2x)$?

9. What are the first few nonzero terms of the Maclaurin series for $f(x) = \frac{1}{2}\sin(2x)$?

9. What are the first few nonzero terms of the Maclaurin series for $f(x) = xe^{-2x^2}$?

(a) $1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \cdots$ (b) $x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{12}x^4 + \cdots$

(c) $x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^4 + \frac{1}{64}x^5 + \cdots$ (d) $x - 2x^3 + \frac{2}{3}x^5 - \frac{4}{45}x^7 + \cdots$

(e) $x - \frac{2}{3}x^3 + \frac{2}{15}x^5 - \frac{4}{315}x^7 + \cdots$ (f) $x - 2x^3 + 2x^5 - \frac{4}{3}x^7 + \cdots$

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10. Which of the following is closest to the value of $\int_0^{0.1} \sqrt{1+x^2}\,dx$? Justify your answer.

10. Which of the following is closest to the value of $\int_0^{0.1} e^{-x^2/2}\,dx$? Justify your answer.

10. Which of the following is closest to the value of $\int_0^{0.1} \cos(x^2)\,dx$? Justify your answer.

(a) 0.0998 (b) 0.0999 (c) 0.1000 (d) 0.1001 (e) 0.1002 (f) 0.1003

MATH 104 – Third Midterm Exam - Fall 2014

1. What is the limit of the sequence defined recursively by $a_1 = 0$ and $a_{n+1} = \sqrt{a_n + 6}$?

1. What is the limit of the sequence defined recursively by $a_1 = 0$ and $a_{n+1} = \sqrt{a_n + 12}$?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 12 (f) 20

1. What is the limit of the sequence defined recursively by $a_1 = 0$ and $a_{n+1} = \sqrt{a_n + 20}$?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 12 (f) 20

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2. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+3}}{n^2 - \sqrt{n} + 5}$ converge or diverge? (Why?)

2. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{n - \sqrt{n} + 4}$ converge or diverge? (Why?)

2. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+5}}{n^3 - \sqrt{n} + 3}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

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3. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 3^n}{n!}$ converge or diverge? (Why?)

3. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^2 3^n}$ converge or diverge? (Why?)

3. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^3}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

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4. Does the series $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n^2}\right)$ converge or diverge? (Why?)

4. Does the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$ converge or diverge? (Why?)

4. Does the series $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$ converge or diverge? (Why?)

(a) Converges (b) Diverges

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5. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{6^{2n+1}(2n+1)!}$ If the series diverges, explain why.

5. If it converges, find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n3^n}$ If the series diverges, explain why.

5. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$ If the series diverges, explain why.

(a) $\ln 3$ (b) $\ln 4 - \ln 3$ (c) e^2 (d) $1/2$ (e) $1/e^2$ (f) Diverges

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6. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+1}$ converge absolutely, converge conditionally, or diverge? (Why?)

6. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2+1)}{n^3}$ converge absolutely, converge conditionally, or diverge? (Why?)

6. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^4+1}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

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7. For which values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(1+n^2)^p}$ converge?

7. For which values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(1+n^3)^p}$ converge?

7. For which values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^{2p}}{(1+n^2)^p}$ converge?

(a) $p > 1$ (b) $p < -1$ (c) $p > 3$ (d) $p > \frac{1}{3}$ (e) $p < 0$ (f) no value of p

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8. For which values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2 2^n}$ converge?

8. For which values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n+1}$ converge?

8. For which values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2 2^n}$ converge?

(a) $-2 < x \leq 2$ (b) $0 < x < 4$ (c) $0 \leq x \leq 4$

(d) $1 \leq x \leq 3$ (e) $1 \leq x < 3$ (f) $-2 \leq x \leq 2$

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9. What are the first few nonzero terms of the Maclaurin series for $f(x) = \arctan(x^2)$?

9. What are the first few nonzero terms of the Maclaurin series for $f(x) = \sin(x^2)$?

9. What are the first few nonzero terms of the Maclaurin series for $f(x) = x^2 \cos(x^2)$?

(a) $1 - 2x^2 + \frac{2}{3}x^4 + \cdots$ (b) $x^2 - \frac{1}{2}x^6 + \frac{1}{24}x^{10} + \cdots$

(c) $x^2 + \frac{1}{4}x^6 + \frac{1}{16}x^{10} + \cdots$ (d) $x^2 - \frac{1}{3}x^6 + \frac{1}{5}x^{10} + \cdots$

(e) $x^2 - \frac{1}{6}x^6 + \frac{1}{120}x^{10} + \cdots$ (f) $x^2 - \frac{1}{6}x^6 + \frac{1}{5}x^{10} + \cdots$

10. Which of the following is closest to the value of $\int_0^{0.1} \sin x \, dx$? Justify your answer.

10. Which of the following is closest to the value of $\int_0^{0.1} \arctan(x) \, dx$? Justify your answer.

10. Which of the following is closest to the value of $\int_0^{0.1} xe^{-x^2} \, dx$? Justify your answer.

(a) 0.004975 (b) 0.004981 (c) 0.004986 (d) 0.004992 (e) 0.004996 (f) 0.004999

MATH 104 – Third Midterm Exam - Fall 2015

1. For each of the following series, say whether it converges or diverges and which test you would use to prove it. (You don't have to give the details of applying the test)

(1.1) $\sum_{n=1}^{\infty} \frac{n}{n^2+4}$

(a) Converges (b) Diverges Test: _____

(1.2) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(a) Converges (b) Diverges Test: _____

(1.3) $\sum_{n=1}^{\infty} \frac{1}{\ln n}$

(a) Converges (b) Diverges Test: _____

(1.4) $\sum_{n=1}^{\infty} \frac{1}{(n!)^2}$

(a) Converges (b) Diverges Test: _____

2. What is the limit of the sequence defined recursively by $a_1 = 1$ and $a_{n+1} = 2 + \dfrac{1}{a_n}$?

2. What is the limit of the sequence defined recursively by $a_1 = 1$ and $a_{n+1} = 4 + \dfrac{1}{a_n}$?

2. What is the limit of the sequence defined recursively by $a_1 = 1$ and $a_{n+1} = 8 + \dfrac{1}{a_n}$?

(a) $1 + \sqrt{2}$ (b) $1 + \sqrt{3}$ (c) $2 + \sqrt{5}$ (d) $4 + \sqrt{2}$ (e) $4 + \sqrt{17}$ (f) $8 + \sqrt{2}$

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3. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^{3/2}}{n^3 - \sqrt{n} + 12}$ converge or diverge? (Why?)

3. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{n - \sqrt{n} + 4}$ converge or diverge? (Why?)

3. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)^{3/2}}{n^2 - \sqrt{n} + 3}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

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4. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ converge or diverge? (Why?)

4. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ converge or diverge? (Why?)

4. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n\, n!}{(2n)!}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

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5. Does the series $\sum_{n=1}^{\infty} \left(e^{1/n} - 1\right)$ converge or diverge? (Why?)

5. Does the series $\sum_{n=1}^{\infty} \left(e^{1/n} - e^{-1/n}\right)$ converge or diverge? (Why?)

5. Does the series $\sum_{n=1}^{\infty} \left(1 - e^{-1/n^2}\right)$ converge or diverge? (Why?)

(a) Converges (b) Diverges

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6. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n}(2n)!}$ If the series diverges, explain why.

6. If it converges, find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n2^n}$ If the series diverges, explain why.

6. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n 2}{n!}$ If the series diverges, explain why.

(a) $\ln 2$ (b) $\ln 3 - \ln 2$ (c) $1/e^2$ (d) $1/2$ (e) $2/e$ (f) Diverges

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7. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 \ln n}{n^3 + 1}$ converge absolutely, converge conditionally, or diverge? (Why?)

7. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2 + \ln n)}{n^3}$ converge absolutely, converge conditionally, or diverge? (Why?)

7. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 \ln n}{n^4 + 1}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

8. For which values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^p}{1+n^2}$ converge?

8. For which values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(1+n^p)^4}$ converge?

8. For which values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^p}{(1+n^2)^p}$ converge?

(a) $p < 1$ (b) $p > 1$ (c) $p > 4$ (d) $p > \frac{1}{4}$ (e) $p < 0$ (f) no value of p

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9. For which values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n\,4^n}$ converge?

9. For which values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{(n+1)\,4^n}$ converge?

9. For which values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+1)^n}{n^2\,4^n}$ converge?

(a) $-3 < x < 5$ (b) $-3 \le x < 5$ (c) $-3 < x \le 5$

(d) $-5 < x \le 3$ (e) $-5 \le x < 3$ (f) $-5 \le x \le 3$

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10. Using the Taylor series for $\sqrt[3]{x}$ centered at $x = 8$, determine which of the following is closest to the value of $\sqrt[3]{8.1}$. Justify your answer.

10. Using the Taylor series for $\sqrt[4]{x}$ centered at $x = 16$, determine which of the following is closest to the value of $\sqrt[4]{16.2}$. Justify your answer.

10. Using the Taylor series for $\sqrt[5]{x}$ centered at $x = 32$, determine which of the following is closest to the value of $\sqrt[5]{32.4}$. Justify your answer.

(a) 2.004 (b) 2.005 (c) 2.006 (d) 2.007 (e) 2.008 (f) 2.009

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11. Which of the following is closest to the value of $\int_0^{0.2} \cos(x^2)\,dx$? Justify your answer.

11. Which of the following is closest to the value of $\int_0^{0.2} e^{-2x^2}\,dx$? Justify your answer.

11. Which of the following is closest to the value of $\int_0^{0.2} \frac{1}{1+x^2}\,dx$? Justify your answer.

(a) 0.195 (b) 0.196 (c) 0.197 (d) 0.198 (e) 0.199 (f) 0.200