MATH 104 – Practice Problems for Exam 2

1. Find the area between:

(a)
$$x = 0$$
, $y = 1/\sqrt{1+x^2}$, $y = x/\sqrt{2}$

Answer:
$$\ln(1+\sqrt{2}) - \frac{\sqrt{2}}{4}$$

(b)
$$y = 3e^{2x}$$
, $y = xe^{2x}$, $x = 0$
Answer: $\frac{e^6}{4} - \frac{7}{4}$

Answer:
$$\frac{e^6}{4} - \frac{7}{4}$$

(c)
$$y = \frac{x}{x^2 - 1}$$
 and the x axis, for $2 \le x \le 4$.

Answer:
$$\frac{\ln 5}{2}$$

2. Calculate the volume obtained by rotating:

(a) The region in problem 1a around the
$$x$$
-axis

Answer:
$$\frac{\pi^2}{4} - \frac{\pi}{6}$$

(b) The region in problem 1a around the
$$y$$
-axis

Answer:
$$2\pi \left(\frac{5\sqrt{2}}{6} - 1\right)$$

(c) The region in problem 1b around the
$$x$$
-axis

Answer:
$$\pi \left(\frac{11e^{12}}{32} - \frac{71}{32} \right)$$

(d) The region in problem 1b around the
$$y$$
-axis

Answer:
$$\pi(e^6+2)$$

Answer:
$$\pi(\frac{1}{2}\ln 3 - \frac{1}{4}\ln 5 + \frac{1}{5})$$

(f) The region in problem 1c around the
$$y$$
-axis

Answer:
$$\pi(2 \ln 3 - \ln 5 + 4)$$

(g) The region in problem 1c around the line
$$x = 1$$

Answer:
$$2\pi(\ln 3 - \ln 5 + 2)$$

(h) The region in problem 1c around the line
$$y = -1$$

Answer:
$$\pi(\frac{1}{2}\ln 3 + \frac{3}{4}\ln 5 + \frac{1}{5})$$

3. Integrate: (straightforward)

(a)
$$\int x^4 e^{2x} dx$$

Answer:
$$\frac{1}{4}e^{2x}(2x^4 - 4x^3 + 6x^2 - 6x + 3) + C$$

(b)
$$\int x^2 \tan^{-1}(x) \, dx$$

Answer:
$$\frac{1}{3}x^3 \arctan x - \frac{1}{6}x^2 + \frac{1}{6}\ln(1+x^2) + C$$

(c)
$$\int \frac{x}{x^2 - 5x + 4} dx$$

(c)
$$\int \frac{x}{x^2 - 5x + 4} dx$$

Answer: $\frac{4}{3} \ln(x - 4) - \frac{1}{3} \ln(x - 1) + C$

(d)
$$\int \sqrt{1+4x^2} \, dx$$

Answer:
$$\frac{1}{2}x\sqrt{1+4x^2} + \frac{1}{4}\ln(2x+\sqrt{1+4x^2}) + C$$

(e)
$$\int \frac{1}{1+\sqrt{x}} \, dx$$

Answer:
$$2\sqrt[3]{x} - 2\ln(1+\sqrt{x}) + C$$

(f)
$$\int \frac{\cos^2 \sqrt{x}}{\sqrt{x}} \, dx$$

Answer:
$$\sqrt{x} + \frac{1}{2}\sin(2\sqrt{x}) + C$$

(g)
$$\int \frac{\sec(\ln x)\tan(\ln x)}{x} dx$$
 Answer: $\sec(\ln x) + C$

Answer:
$$\sec(\ln x) + C$$

4. Integrate: (trickier)

(a)
$$\int \sin^4(2x) \, dx$$

Answer:
$$\frac{3}{8}x - \frac{3}{16}\cos(2x)\sin(2x) - \frac{1}{8}\cos(2x)\sin^3(2x) + C$$

(b)
$$\int \frac{\sqrt{x^2 - 25}}{x} dx$$

Answer:
$$\sqrt[3]{x^2 - 25} + 5\arcsin(5/x) + C$$
 (Note that $\arcsin(5/x) = \pi - \operatorname{arcsec}(x/5)$ [Why?])

(c)
$$\int \frac{e^t}{e^{2t} - 4} dt$$

Answer:
$$\frac{1}{4}(\ln(e^t - 2) - \ln(e^t + 2)) + C$$

(d)
$$\int \sqrt{1+e^x} \, dx$$

Answer:
$$2\sqrt{1+e^x} + \ln(\sqrt{1+e^x} - 1) - \ln(\sqrt{1+e^x} + 1) + C$$

(e)
$$\int e^{\sqrt{x}} dx$$

Answer:
$$2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

5. Evaluate:

(a)
$$\int_{e}^{\infty} \frac{1}{x(\ln x)^3} \, dx$$

Answer:
$$1/2$$

(b)
$$\int_0^\infty \frac{dx}{\sqrt{x}(x+4)}$$

(c)
$$\int_0^1 \sqrt{\frac{1-y}{y}} \, dy$$

Answer: $\pi/2$

6. Find the general solution to each of the following differential equations:

(a)
$$x \frac{dy}{dx} = y^2$$

(a) $x \frac{dy}{dx} = y^2$ Answer: $y = 1/(C - \ln(x))$

(b)
$$(x^2 + 1)\frac{dy}{dx} = y$$

Answer: $y = Ce^{\arctan x}$

7. Find the specific solution of each equation that satisfies the given condition:

(a)
$$\frac{dy}{dx} = xy$$
, $y(1) = 3$
Answer: $y = 3e^{(x^2-1)/2}$

(b)
$$\frac{dy}{dx} = xy + x, \quad y(0) = 10$$

Answer $y = 11e^{x^2/2} - 1$

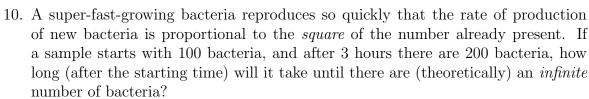
8. In a second-order chemical reaction, the reactant A is used up in such a way that the amount of it present decreases at a rate proportional to the square of the amount present. Suppose this reaction begins with 50 grams of A present, and after 10 seconds there are only 25 grams left. How long after the beginning of the reaction will there be only 10 grams left? Will all of the A disappear in a finite time, or will there always be a little bit present?

Answer: 40 seconds, and there will always be a little bit present.

9. According to Newton's law of heating and cooling, if the temperature of an object is different from the temperature of its environment, then the temperature of the object will change so that the difference between the object's temperature and the ambient temperature decreases at a rate proportional to this difference.

On a hot day, a thermometer was brought outdoors from an air-conditioned building. The temperature inside the building was 21° C, and so this is what the thermometer read at the moment it was brought outside. One minute later the thermometer read 27° C, and a minute after that it read 31° C. What was the temperature outside? (Impress us and express the answer without using logarithms or the number e.)

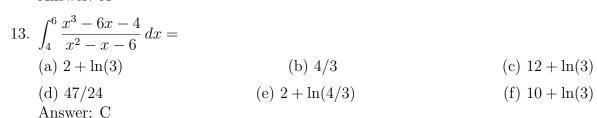
Answer: 39° C



Answer: 6 hours

11.
$$\int_{0}^{1} \frac{x^{2}}{\sqrt{1-x^{2}}} dx =$$
(a) $\frac{2\pi}{15}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{5}$ (d) $\frac{\pi}{2}$ (e) $\frac{3\pi}{5}$ (f) 2π

12.
$$\int_0^\infty x^2 e^{-2x} dx =$$
 (a) 1/4 (b) 4/3 (c) 2 (d) 3/8 (e) $e^{1/2}$ (f) diverges Answer: A



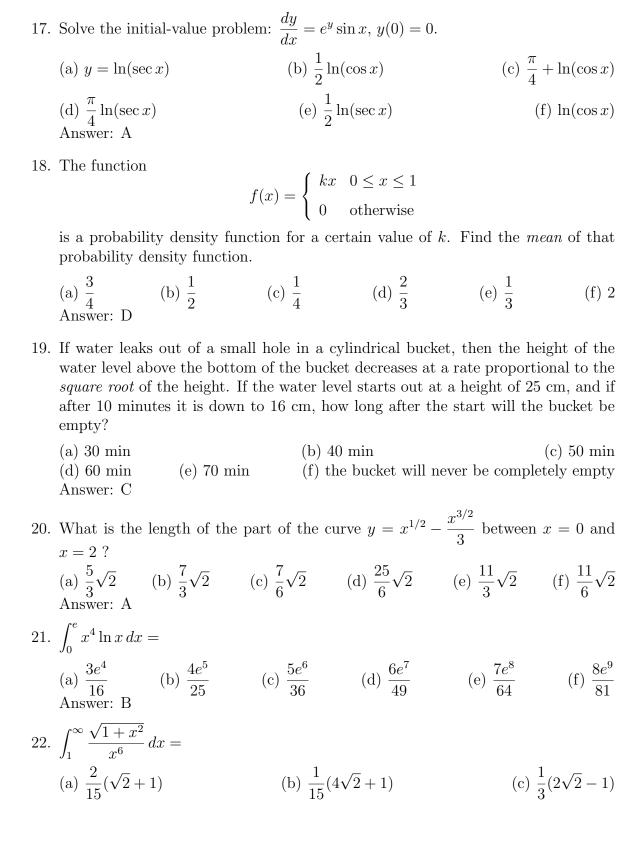
14. What is the volume of the solid obtained by rotating the region between the graph of $y = \frac{1}{x^2 + 4x + 3}$ and the x-axis for $0 \le x \le 1$ around the y-axis?

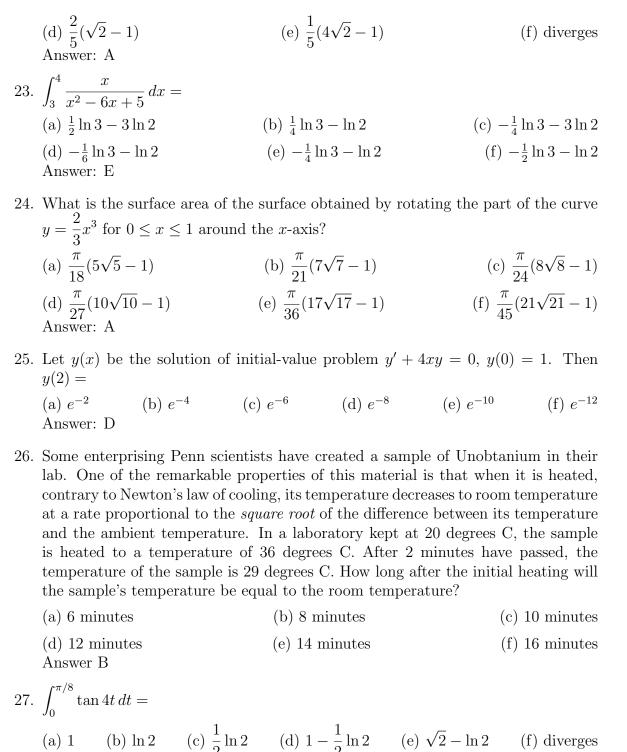
(a)
$$\pi(\ln 2 + 2 \ln 3)$$
 (b) $2\pi(4 \ln 3 - 5 \ln 2)$ (c) $2\pi \ln 12$ (d) $\pi(2 \ln 3 + 3 \ln 2)$ (e) $2\pi \ln 18$ (f) $\pi(5 \ln 2 - 3 \ln 3)$ Answer: F

15.
$$\int_0^\infty \frac{1}{x^2 + 2x + 2} dx =$$
(a) 1 (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$ (e) $\frac{\pi}{2} - 1$ (f) diverges Answer: D

16. Find the surface area of the surface obtained by revolving the part of the graph of $y = x^3/9$ where $0 \le x \le 2$ around the x-axis.

(a)
$$\frac{38\pi}{27}$$
 (b) $\frac{121\pi}{72}$ (c) $\frac{76\pi}{9}$ (d) $\frac{77\pi}{48}$ (e) $\frac{98\pi}{81}$ (f) $\frac{86\pi}{27}$ Answer: E





(a) 1

Answer: F

28. The function

$$f(x) = \begin{cases} kxe^{-4x} & x \ge 0\\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for a certain value of k. Find the mean of that probability density function.

(a) 1

(b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{5}$ (e) $\frac{1}{3}$

Answer: C

29. The functions $y_1(t)$ and $y_2(t)$ are both solutions of the autonomous differential equation $\frac{dy}{dt} = 3\sin\left(\frac{y}{2}\right)$ but satisfy different initial conditions: $y_1(0) = 1$ and $y_2(0) = -1$. Either by solving the differential equation or, better, by thinking about its geometry (slope field), calculate

$$\lim_{t\to\infty}(y_1(t)-y_2(t)).$$

(a) 0 Answer: C (b) 2π

(c) 4π

(d) 6π

(e) 8π

(f) ∞

MATH 104 - Second Midterm Exam - Fall 2014

1. $\int_0^{\pi} \sin^3 x \cos^2 x \, dx$

(a) $\frac{4}{63}$ (b) $\frac{4}{35}$ (c) $\frac{2}{15}$ (d) $\frac{4}{15}$ (e) $\frac{1}{12}$ (f) $\frac{1}{6}$

2. $\int_0^{\pi/2} x \sin(2x) dx$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) $-\frac{\pi}{4}$ (e) $-\frac{\pi}{2}$ (f) $-\pi$ Answer: A

3. $\int_0^2 \frac{x^2}{9-x^2} dx$

(a) $2 \ln 7 - 3$ (b) $2 \ln 9 - 5$ (c) $\frac{3}{2} \ln 5 - 2$ (d) $4 \ln 3 - 5$ (e) $5 \ln 3 - 4$ (f) $\ln 3 - 1$ Answer: C

$$4. \int_0^{\sqrt{3}/2} \arcsin x \, dx$$

(a)
$$\frac{\pi}{2\sqrt{3}} - \frac{1}{2}$$

(b)
$$\frac{\pi}{\sqrt{3}} - \ln 2$$

(c)
$$\frac{\pi}{\sqrt{3}} - \frac{\ln 2}{4}$$

(d)
$$\frac{\pi}{6\sqrt{3}} - \frac{1}{2} \ln \left(\frac{4}{3}\right)$$

(e)
$$\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

(f)
$$\frac{\pi}{6} - \frac{\sqrt{3}}{2}$$

5.
$$\int_{-\infty}^{0} \frac{e^x}{(1+e^{2x})^{3/2}} dx$$

(a)
$$\frac{3\sqrt{2}}{8}$$
 (b) $\frac{5\sqrt{2}}{12}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{5\sqrt{2}}{24}$ (e) $\frac{5\sqrt{2}}{6}$ (f) $\frac{\sqrt{2}}{4}$

(b)
$$\frac{5\sqrt{2}}{12}$$

(c)
$$\frac{\sqrt{2}}{2}$$

(d)
$$\frac{5\sqrt{2}}{24}$$

(e)
$$\frac{5\sqrt{2}}{6}$$

(f)
$$\frac{\sqrt{2}}{4}$$

$$f(x) = \begin{cases} \frac{k}{x^6} & x \ge 1\\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for a certain value of k. Find the mean of that probability density function.

(a)
$$\frac{5}{4}$$
 (b) $\frac{5}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{2}$ (e) $\frac{3}{2}$

(b)
$$\frac{5}{3}$$

(c)
$$\frac{4}{3}$$

(d)
$$\frac{5}{2}$$

(e)
$$\frac{3}{2}$$

7. The solution of the initial-value problem

$$x\frac{dy}{dx} + 5y = 6x \qquad y(1) = 1$$

satisfies y(2) =

$$(d)$$
 6

- Answer: A
- 8. On a cold winter day, when the temperature outside is 10 degrees, Bart finds his skateboard on the roof and brings it indoors, where the temperature is 70 degrees. After being indoors for 20 minutes, the temperature of the skateboard rises to 34 degrees. What will the temperature of the skateboard be after another 20 minutes (i.e., 40 minutes after being brought indoors)? Assume Newton's law of cooling (and heating) applies.

(a) 31.6 degrees

(b) 40.2 degrees

(c) 48.4 degrees

(d) 56.8 degrees

(e) 60.4 degrees

(f) 67.6 degrees

Answer: C

- **9**. The region between the x-axis and the graph of $y = \sin \frac{x}{2}$ for $0 \le x \le 2\pi$ is rotated around the x-axis to generate a solid. What is the volume of the solid?
 - (a) $\frac{\pi^2}{2}$ (b) π^2 (c) $2\pi^2$ (d) $4\pi^2$ (e) $8\pi^2$

- (f) $16\pi^2$

Answer: B

- 10. Tank number 1 holds 100 liters of water in which 50 kg of salt is initially dissolved. At time t=0, pure water begins to flow into tank number 1 at a rate of 2 liters per minute and the well-stirred mixture flows out at the same rate, into a second tank, which initially contains 100 liters of pure water. The well-stirred mixture in the second tank also flows out at the same rate. If S(t) is the amount of salt (in kg) in the second tank at time t (minutes), what is the differential equation satisfied by S?

- (a) $S' = e^{-t/50} \frac{1}{50}S$ (b) $S' = \frac{1}{2}e^{-t/50} \frac{1}{100}S$ (c) $S' = e^{-t/100} \frac{1}{100}S$ (d) $S' = e^{-t/50} \frac{1}{100}S$ (e) $S' = \frac{1}{2}e^{-t/50} \frac{1}{50}S$ (f) $S' = \frac{1}{2}e^{-t/100} \frac{1}{50}S$

Answer: A

MATH 104 – Second Midterm Exam - Fall 2015

- 1. $\int_0^{\pi/4} \tan^2 x \sec^4 x \, dx$
- (a) $\frac{4}{63}$ (b) $\frac{7}{24}$ (c) $\frac{5}{12}$ (d) $\frac{8}{15}$ (e) $\frac{12}{35}$

- 2. $\int_0^{\ln 2} x e^{-2x} dx$
- (a) $\frac{3}{16} \frac{1}{8} \ln 2$

(b) $-\frac{3}{8} + \frac{2}{3} \ln 2$

(c) $\frac{5}{24} - \frac{1}{6} \ln 2$

(d) $\frac{7}{72} - \frac{1}{24} \ln 2$

(e) $-\frac{7}{9} + \frac{8}{3} \ln 2$

(f) $-\frac{3}{4} + 2 \ln 2$

$$3. \int_0^1 \frac{16x^2 + 2x + 3}{(x+1)(16x^2 + 1)} \, dx$$

(a) $3 \ln 2 + \arctan 4$

(b) $\ln 2 + \frac{1}{4} \arctan 4$

(c) $2 \ln 2 + \frac{3}{2} \arctan 2$

(d) $4 \ln 2 - \frac{1}{2} \arctan 2$

Answer: E

(e) $\ln 2 + \frac{1}{2} \arctan 4$

(f) $\ln 2 - \arctan 2$

4. $\int_{1}^{e^2} 9 x^2 \ln x \, dx$

(a) $1 + 7e^8$

(b) $1 + 9e^{10}$

(c) $3 + 5e^8$

(d) $1 + 5e^6$

(e) $4 + 8e^3$

(f) $2 + 7e^6$

Answer: D

$$5. \int_{1}^{\infty} \frac{12}{x(9 + (\ln x)^2)^{3/2}} \, dx$$

(a) 1

(b) 2

(c) $\frac{1}{4}$ (d) $\frac{4}{3}$

(e) $\frac{5}{4}$

6. The function

Answer: D

$$f(x) = \begin{cases} \frac{k}{x^{2/3}} & 0 < x \le 1\\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for a certain value of k. Find the mean of that probability density function.

(a) $\frac{1}{5}$ Answer: C

(b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$ (e) $\frac{2}{5}$

(f) $\frac{1}{3}$

7. The solution of the initial-value problem

 $\frac{dy}{dx} = -4x\sqrt{y} \qquad y(0) = 4$

satisfies y(1) =

(a) 0

(b) 1

(c) 2

(d) 4

(e) 9

(f) 12

Answer: B

8. In a second-order chemical reaction, the reactant R is used up in such a way that the amount of it present decreases at a rate proportional to the square of the amount present. Suppose this reaction begins with 200 grams of R present, and after 12 seconds

there are only	100 grams left.	How long after	the beginning	of the reaction	on will there be
only 40 grams	left?				

(a) 40 seconds

(b) 45 seconds

(c) 48 seconds

(d) 50 seconds

(e) 64 seconds

(f) 80 seconds

Answer: C

9. The region between the x-axis and the graph of $y = \cos \frac{x}{8}$ for $-4\pi \le x \le 4\pi$ is rotated

around the x-axis to generate a solid. What is the volume of the solid?

(a) $\frac{\pi^2}{2}$

(b) π^2

(c) $2\pi^2$

(d) $4\pi^2$

(e) $8\pi^2$

(f) $16\pi^2$

Answer: D

10. Consider the initial-value problem: $y' - 2y = 15 \sin x$ y(0) = A. For which value of the constant A will the solution be *periodic* (with period 2π)?

(a) A = -4

(b) A = 4

(c) A = -1

(d) A = 1

(e) A = -3

(f) A = 3

Answer: E