## MATH 104 – First Midterm Exam - Fall 2016

- 1. Find the area between the graphs of  $y = 2x^2$  and  $y = x^3$ .
- 1. Find the area between the graphs of  $y = 3x^2$  and  $y = x^3$ .
- 1. Find the area between the graphs of  $y = 4x^2$  and  $y = x^3$ .
- 1. Find the area between the graphs of  $y = 6x^2$  and  $y = x^3$
- (a)  $\frac{4}{3}$

- (b)  $\frac{27}{4}$  (c)  $\frac{33}{4}$  (d)  $\frac{27}{2}$  (e)  $\frac{64}{3}$

(f) 108

- A solid has as its base the region in the xy-plane the region between the curve  $y=1-\frac{x^2}{4}$  and the x-axis. The cross-sections of the solid perpendicular to the x-axis are isosceles right triangles (i.e., 45-45-90 triangles) with the longest side (i.e., the hypotenuse) in the base. What is the volume of the solid?
- 2. A solid has as its base the region in the xy-plane the region between the curve  $y = 1 - \frac{x^2}{16}$  and the x-axis. The cross-sections of the solid perpendicular to the x-axis are isosceles right triangles (i.e., 45-45-90 triangles) with the longest side (i.e., the hypotenuse) in the base. What is the volume of the solid?
- 2. A solid has as its base the region in the xy-plane the region between the curve  $y=1-\frac{x^2}{Q}$  and the x-axis. The cross-sections of the solid perpendicular to the x-axis are isosceles right triangles (i.e., 45-45-90 triangles) with the longest side (i.e., the hypotenuse) in the base. What is the volume of the solid?
- 2. A solid has as its base the region in the xy-plane the region between the curve  $y = 1 - \frac{x^2}{36}$  and the x-axis. The cross-sections of the solid perpendicular to the x-axis are isosceles right triangles (i.e., 45-45-90 triangles) with the longest side (i.e., the hypotenuse) in the base. What is the volume of the solid?
- (a)  $\frac{4}{15}$  (b)  $\frac{8}{15}$  (c)  $\frac{4}{5}$  (d)  $\frac{16}{15}$  (e)  $\frac{8}{5}$  (f)  $\frac{16}{5}$

3. Let $V(a)$ be the volume of the solid obtained by rotating the area between the graph of $y=x^{-3/2}$ and the x-axis for $a < x < 1$ around the y-axis. What is $\lim_{a \to 0+} V(a)$ ?
3. Let $V(a)$ be the volume of the solid obtained by rotating the area between the graph of $y = x^{-4/3}$ and the x-axis for $a < x < 1$ around the y-axis. What is $\lim_{a \to 0+} V(a)$ ?

**3**. Let V(a) be the volume of the solid obtained by rotating the area between the graph of  $y=x^{-5/3}$  and the x-axis for a < x < 1 around the y-axis. What is  $\lim_{a \to 0+} V(a)$ ?

**3**. Let V(a) be the volume of the solid obtained by rotating the area between the graph of  $y=x^{-7/4}$  and the x-axis for a < x < 1 around the y-axis. What is  $\lim_{a \to 0+} V(a)$ ?

- (a)  $\pi$  (b)  $2\pi$  (c)  $3\pi$  (d)  $4\pi$  (e)  $6\pi$  (f)  $8\pi$
- **4.** Find the length of the part of the curve  $y = \frac{x^3}{6} + \frac{1}{2x}$  for  $1 \le x \le 2$ .
- **4**. Find the length of the part of the curve  $y = \frac{x^3}{4} + \frac{1}{3x}$  for  $1 \le x \le 2$ .
- **4**. Find the length of the part of the curve  $y = \frac{x^3}{24} + \frac{2}{x}$  for  $1 \le x \le 2$ .
- **4**. Find the length of the part of the curve  $y = \frac{x^3}{8} + \frac{2}{3x}$  for  $1 \le x \le 2$ .
- (a)  $\frac{29}{24}$  (b)  $\frac{31}{24}$  (c)  $\frac{17}{12}$  (d)  $\frac{19}{12}$  (e)  $\frac{39}{24}$  (f)  $\frac{23}{12}$

5. 
$$\int_{1}^{e^2} \frac{\sin(\pi \ln x)}{x} dx$$

$$5. \int_1^e \frac{\sin(\pi \ln x)}{x} \, dx$$

5. 
$$\int_{1}^{e^{1/2}} \frac{\sin(\pi \ln x)}{x} dx$$

5. 
$$\int_{1}^{e^{1/3}} \frac{\sin(\pi \ln x)}{x} dx$$

- (a)  $\pi$
- (b)  $2\pi$

- (c)  $\frac{2}{\pi}$  (d)  $\frac{1}{\pi}$  (e)  $\frac{1}{2\pi}$
- (f) 0
- **6.** An object moves in such a way that its velocity at time t seconds is  $\frac{1}{\pi(1+4t^2)}$  meters per second. If the object starts at the origin, and x(t) is the position of the object at time t seconds, what is the limit of the position as  $t \to \infty$ ?
- **6**. An object moves in such a way that its velocity at time t seconds is  $\frac{1}{\pi(1+9t^2)}$  meters per second. If the object starts at the origin, and x(t) is the position of the object at time t seconds, what is the limit of the position as  $t \to \infty$ ?
- **6**. An object moves in such a way that its velocity at time t seconds is  $\frac{1}{\pi(1+16t^2)}$ meters per second. If the object starts at the origin, and x(t) is the position of the object at time t seconds, what is the limit of the position as  $t \to \infty$ ?
- **6**. An object moves in such a way that its velocity at time t seconds is  $\frac{1}{\pi(1+25t^2)}$ meters per second. If the object starts at the origin, and x(t) is the position of the object at time t seconds, what is the limit of the position as  $t \to \infty$ ?
- (a)  $\frac{1}{2}$  m

(b)  $\frac{1}{4}$  m

(c)  $\frac{1}{6}$  m

(d)  $\frac{1}{8}$  m

(e)  $\frac{1}{10}$  m

(f)  $\frac{1}{12}$  m

7. Find the surface area obtained by rotating the part of the curve  $y = \frac{x^3}{3}$  for  $0 < x < \sqrt{2}$  around the x-axis.

Find the surface area obtained by rotating the part of the curve  $y = \frac{x^3}{3}$  for  $0 < x < \sqrt{3}$  around the x-axis.

7. Find the surface area obtained by rotating the part of the curve  $y = \frac{x^3}{3}$  for 0 < x < 2around the x-axis.

Find the surface area obtained by rotating the part of the curve  $y = \frac{x^3}{3}$  for  $0 < x < \sqrt{5}$  around the x-axis.

(a) 
$$\frac{\pi}{9}(5\sqrt{5}-1)$$

(b) 
$$\frac{\pi}{9}(7\sqrt{7}-1)$$

(c) 
$$\frac{\pi}{9}(10\sqrt{10}-1)$$

(d) 
$$\frac{\pi}{9}(13\sqrt{13}-1)$$

(e) 
$$\frac{\pi}{9}(17\sqrt{17}-1)$$

(f) 
$$\frac{\pi}{9}(26\sqrt{26}-1)$$

8. Let X(b) be the x-coordinate of the centroid of the region bounded by the graph of  $y = \frac{1}{x^3}$  and the x-axis for  $1 \le x \le b$ . What is  $\lim_{b \to \infty} X(b)$ ?

8. Let X(b) be the x-coordinate of the centroid of the region bounded by the graph of

 $y = \frac{1}{x^4}$  and the x-axis for  $1 \le x \le b$ . What is  $\lim_{b \to \infty} X(b)$ ?

8. Let X(b) be the x-coordinate of the centroid of the region bounded by the graph of

 $y = \frac{1}{x^5}$  and the x-axis for  $1 \le x \le b$ . What is  $\lim_{b \to \infty} X(b)$ ?

8. Let X(b) be the x-coordinate of the centroid of the region bounded by the graph of

 $y = \frac{1}{x^6} \text{ and the } x\text{-axis for } 1 \le x \le b. \text{ What is } \lim_{b \to \infty} X(b)?$ (a) 2 (b)  $\frac{3}{2}$  (c)  $\frac{4}{3}$  (d)  $\frac{5}{4}$ 

(e)  $\frac{6}{5}$ 

(f)  $\frac{7}{6}$ 

- **9**. The region between the graph of  $x = 1 y^2$  and the y-axis is rotated around the line  $y = \frac{3}{2}$ . What is the volume of the resulting solid?
- **9**. The region between the graph of  $x = 1 y^2$  and the y-axis is rotated around the line y = 3. What is the volume of the resulting solid?
- 9. The region between the graph of  $x = 1 y^2$  and the y-axis is rotated around the line  $y = \frac{9}{2}$ . What is the volume of the resulting solid?
- **9**. The region between the graph of  $x = 1 y^2$  and the y-axis is rotated around the line y = 6. What is the volume of the resulting solid?
- (a)  $2\pi$  (b)  $4\pi$  (c)  $8\pi$  (d)  $12\pi$  (e)  $15\pi$  (f)  $16\pi$
- 10. Suppose the region R between the graphs of y = f(x) and y = g(x) is contained between the y-axis and the line x = 2. Further, suppose that the area of the region R is A, and the volume of the solid obtained by rotating the region R around the y-axis is V. What is the volume of the solid obtained by rotating the region R around the line x = 4?

(*Hint:* Draw a picture, write the expression for the volume of the latter solid and reexpress it terms of A and V.)

10. Suppose the region R between the graphs of y = f(x) and y = g(x) is contained between the y-axis and the line x = 2. Further, suppose that the area of the region R is A, and the volume of the solid obtained by rotating the region R around the y-axis is V. What is the volume of the solid obtained by rotating the region R around the line x = 8?

(*Hint:* Draw a picture, write the expression for the volume of the latter solid and reexpress it terms of A and V.)

10. Suppose the region R between the graphs of y = f(x) and y = g(x) is contained between the y-axis and the line x = 2. Further, suppose that the area of the region R is A, and the volume of the solid obtained by rotating the region R around the y-axis is V. What is the volume of the solid obtained by rotating the region R around the line x = 16?

(*Hint:* Draw a picture, write the expression for the volume of the latter solid and reexpress it terms of A and V.)

10. Suppose the region R between the graphs of y = f(x) and y = g(x) is contained between the y-axis and the line x = 2. Further, suppose that the area of the region R is

A, and the volume of the solid obtained by rotating the region R around the y-axis is V. What is the volume of the solid obtained by rotating the region R around the line x=32?

(*Hint:* Draw a picture, write the expression for the volume of the latter solid and reexpress it terms of A and V.)

(a) 
$$2\pi A - V$$

(b) 
$$4\pi A - V$$

(c) 
$$8\pi A - V$$

(d) 
$$16\pi A - V$$

(e) 
$$32\pi A - V$$

(f) 
$$64\pi A - V$$

## MATH 104 - Second Midterm Exam - Fall 2016

- 1.  $\int_0^{\pi/4} 70 \cos^4 2x \sin^3 2x \, dx$
- $1. \int_0^{\pi/4} 45 \cos^2 4x \, \sin^3 4x \, dx$
- 1.  $\int_0^{\pi/4} 96 \cos^3 2x \sin^3 2x \, dx$
- 1.  $\int_0^{\pi/4} 63\cos^6 4x \sin^3 4x \, dx$
- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) 4
- (f) 6

- **2**.  $\int_{1}^{e^2} 16x^3 \ln x \, dx$
- 2.  $\int_{1}^{e^2} 9x^2 \ln x \, dx$
- $2. \int_1^{e^2} \frac{\ln x}{x^2} \, dx$
- 2.  $\int_{1}^{e^2} \frac{4 \ln x}{x^3} dx$
- (a)  $1 + 7e^8$

(b)  $1 - 7e^{-6}$ 

(c)  $1 + 3e^4$ 

(d)  $1 - 3e^{-2}$ 

(e)  $1 - 5e^{-4}$ 

(f)  $1 + 5e^6$ 

- $3. \int_{10}^{\infty} \frac{4}{x^2 4} \, dx$
- $3. \int_{12}^{\infty} \frac{6}{x^2 9} \, dx$
- $3. \int_{16}^{\infty} \frac{4}{x^2 4} \, dx$
- $3. \int_{18}^{\infty} \frac{6}{x^2 9} \, dx$
- (a)  $2 \ln 3 \ln 7$

(b)  $\ln 2 - \ln 5$ 

(c)  $\ln 3 - \ln 2$ 

(d)  $\ln 5 - \ln 3$ 

(e)  $\ln 7 - \ln 5$ 

(f)  $\infty$ 

4. 
$$\int_0^{\pi/4} 64x^2 \sin 2x \, dx$$

4. 
$$\int_0^{\pi/4} 64x^2 \cos 2x \, dx$$

4. 
$$\int_0^{\pi/4} 64x^2 \cos 4x \, dx$$

4. 
$$\int_0^{\pi/4} 64x^2 \cos 4x \, dx$$

(a) 
$$8\pi - 16$$

(b) 
$$2\pi^2 - 16$$

(c) 
$$4\pi^2 - 8$$

(d) 
$$-2\pi$$

(e) 
$$\pi^2 - 4$$

(f) 
$$-4\pi^2$$

5. 
$$\int_0^{\sqrt{1/2}} 96x \sqrt{1-x^4} \, dx$$

$$5. \int_0^{\sqrt{2}} 96x \sqrt{16 - x^4} \, dx$$

$$5. \int_0^{\sqrt{1/8}} 96x \sqrt{1 - 16x^4} \, dx$$

$$5. \int_0^{\sqrt{2}} 96x \sqrt{1 - \frac{x^4}{16}} \, dx$$

(a) 
$$24\pi + 36\sqrt{3}$$

(b) 
$$4\pi + 6\sqrt{3}$$

(c) 
$$64\pi + 96\sqrt{3}$$

(d) 
$$\pi + \frac{3\sqrt{3}}{2}$$

(e) 
$$2\pi + 3\sqrt{3}$$

(f) 
$$16\pi + 24\sqrt{3}$$

**6**. For what values of the constants A and B is the function

$$f(x) = \begin{cases} \frac{A}{x^B} & x > 1\\ 0 & \text{otherwise} \end{cases}$$

is a probability density function with mean  $\frac{4}{3}$ . What are A and B?

**6**. For what values of the constants A and B is the function

$$f(x) = \begin{cases} \frac{A}{x^B} & x > 1\\ 0 & \text{otherwise} \end{cases}$$

is a probability density function with mean  $\frac{6}{5}$ . What are A and B?

**6**. For what values of the constants A and B is the function

$$f(x) = \begin{cases} \frac{A}{x^B} & x > 1\\ 0 & \text{otherwise} \end{cases}$$

is a probability density function with mean  $\frac{8}{7}$ . What are A and B?

**6**. For what values of the constants A and B is the function

$$f(x) = \begin{cases} \frac{A}{x^B} & x > 1\\ 0 & \text{otherwise} \end{cases}$$

is a probability density function with mean  $\frac{9}{8}$ . What are A and B?

(a) 
$$A = 4$$
 and  $B = 5$ 

(b) 
$$A = 5$$
 and  $B = 6$ 

(c) 
$$A = 6$$
 and  $B = 7$ 

(d) 
$$A = 7 \text{ and } B = 8$$

(e) 
$$A = 8$$
 and  $B = 9$ 

(f) 
$$A = 9$$
 and  $B = 10$ 

 $xy' - 3y = x^4 e^x$ 7. The solution of the initial-value problem y(1) = esatisfies y(2) =

 $xy' - 4y = x^5 e^x$ y(1) = e7. The solution of the initial-value problem satisfies y(2) =

 $xy' - 2y = x^3 e^x \qquad y(1) = e$ 7. The solution of the initial-value problem satisfies y(2) =

7. The solution of the initial-value problem  $xy' - 5y = x^6 e^x$   $y(1) = e^x$ satisfies y(2) =

(a) 
$$e^2$$

(b) 
$$2e^2$$

(c) 
$$4e^2$$

(d) 
$$8e^2$$

(d) 
$$8e^2$$
 (e)  $16e^2$ 

8. In a chemical reaction in which three molecules must come together at once, the reactant R is used up in such a way that the amount of it present decreases at a rate proportional to the *cube* of the amount present. Suppose this reaction begins with  $\frac{1}{2}$ gram of R present, and after 8 seconds there is only  $\frac{1}{4}$  gram left. How long after the beginning of the reaction will there be only  $\frac{1}{10}$  gram left?

8. In a chemical reaction in which three molecules must come together at once, the reactant R is used up in such a way that the amount of it present decreases at a rate proportional to the *cube* of the amount present. Suppose this reaction begins with  $\frac{1}{2}$ gram of R present, and after 12 seconds there is only  $\frac{1}{4}$  gram left. How long after the beginning of the reaction will there be only  $\frac{1}{10}$  gram left?

8. In a chemical reaction in which three molecules must come together at once, the reactant R is used up in such a way that the amount of it present decreases at a rate proportional to the *cube* of the amount present. Suppose this reaction begins with  $\frac{1}{2}$ gram of R present, and after 16 seconds there is only  $\frac{1}{4}$  gram left. How long after the beginning of the reaction will there be only  $\frac{1}{10}$  gram left?

8. In a chemical reaction in which three molecules must come together at once, the reactant R is used up in such a way that the amount of it present decreases at a rate proportional to the *cube* of the amount present. Suppose this reaction begins with  $\frac{1}{2}$ gram of R present, and after 20 seconds there is only  $\frac{1}{4}$  gram left. How long after the beginning of the reaction will there be only  $\frac{1}{10}$  gram left?

(a) 64 seconds

(b) 72 seconds

(c) 96 seconds

(d) 108 seconds

(e) 128 seconds

(f) 160 seconds

**9.**  $\int_0^{1/4} e^{2\sqrt{x}} dx$ 

**9.**  $\int_0^{1/9} e^{3\sqrt{x}} dx$ 

**9**.  $\int_0^4 e^{\sqrt{x}/2} dx$ 

**9.**  $\int_0^9 e^{\sqrt{x}/3} dx$ 

(a)  $\frac{1}{2}$  (b)  $\frac{2}{9}$ 

(c)  $\frac{2}{3}$ 

(d) 8

(e) 12

(f) 18

$$10. \int_0^\infty \frac{16}{x^2 + 4x + 8} \, dx$$

$$10. \int_{-4}^{\infty} \frac{16}{x^2 + 8x + 20} \, dx$$

$$10. \int_{2}^{\infty} \frac{16}{x^2 + 4x + 20} \, dx$$

$$10. \int_{-7}^{\infty} \frac{16}{x^2 + 6x + 25} \, dx$$

(a)  $\pi$  (b)  $2\pi$  (c)  $3\pi$  (d)  $4\pi$  (e)  $5\pi$  (f)  $6\pi$ 

1. (Counts as two problems.) For each of the following series, say whether it converges or diverges and which test you would use to prove it. (You don't have to give the details of applying the test)

$$(1.1) \sum_{n=1}^{\infty} \frac{1}{n^2 + 6}$$

$$(1.1) \sum_{n=1}^{\infty} \frac{n^2}{n^4 + 2}$$

$$(1.1) \sum_{n=1}^{\infty} \frac{n^2}{n^3 + 4}$$

(a) Converges

(b) Diverges

Test: \_

$$(1.2) \sum_{n=1}^{\infty} \frac{3^{n+1}}{n!}$$

$$(1.2) \sum_{n=1}^{\infty} \frac{n!}{3^{n+1}}$$

$$(1.2) \sum_{n=1}^{\infty} \frac{2^n}{1+3^n}$$

(b) Diverges

Test: \_

(a) Converges (1.3) 
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{1+n}}$$

$$(1.3) \sum_{n=1}^{\infty} \frac{1}{\sqrt{2 + \cos n}}$$

$$(1.3) \sum_{n=1}^{\infty} \frac{1}{\arctan \sqrt{n}}$$

(a) Converges

(b) Diverges

Test: \_

$$(1.4) \sum_{n=1}^{\infty} \frac{1}{n^n}$$

$$(1.4) \sum_{n=1}^{\infty} \frac{1}{\sqrt{1+n!}}$$

$$(1.4) \sum_{n=1}^{\infty} \arctan(n)$$

- (a) Converges
- (b) Diverges

Test:

- **2.** What is the limit of the sequence  $\left\{\arctan\left(\frac{2n}{2n+1}\right)\right\}$ ?
- **2**. What is the limit of the sequence  $\left\{\arcsin\left(\frac{2n}{2n+1}\right)\right\}$ ?
- **2**. What is the limit of the sequence  $\left\{\arccos\left(\frac{2n}{2n+1}\right)\right\}$ ?
- (a)  $\frac{\pi}{4}$  (b)  $\frac{\pi}{3}$  (c)  $\frac{\pi}{2}$
- (d)  $\pi$

(e) 0

- (f) The sequence diverges.
- **3**. Does the series  $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{n^5-3n^2+20}$  converge or diverge? (Why?)
- 3. Does the series  $\sum_{n=1}^{\infty} \frac{n^3+1}{\sqrt{n^5-3n^2+20}}$  converge or diverge? (Why?)
- 3. Does the series  $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{n^3+2}$  converge or diverge? (Why?)
- (a) Converges

(b) Diverges

- **4.** Does the series  $\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n!}}$  converge or diverge? (Why?)
- **4**. Does the series  $\sum_{n=1}^{\infty} \frac{\sqrt{n!}}{2^n}$  converge or diverge? (Why?)
- 4. Does the series  $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^n}$  converge or diverge? (Why?)
- (a) Converges

(b) Diverges

- 5. Does the series  $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n}$  converge or diverge? (Why?)
- **5**. Does the series  $\sum_{n=1}^{\infty} \frac{\ln(1/n)}{n}$  converge or diverge? (Why?)
- 5. Does the series  $\sum_{n=1}^{\infty} \frac{\arctan(1/n)}{n}$  converge or diverge? (Why?)
- (a) Converges (b) Diverges
- **6.** If it converges, find the sum of the series  $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{6^{2n+1}(2n+1)!}$  If the series diverges, explain why.
- **6**. If it converges, find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n3^n}$  If the series diverges, explain why.
- **6**. If it converges, find the sum of the series  $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$  If the series diverges, explain why.
- (a) ln 3
- (b)  $2 \ln 2 \ln 3$
- (c) 1/e
- (d) 1/2
- (e) e
- (f) Diverges
- 7. Does the series  $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1}$  converge absolutely, converge conditionally, or diverge? (Why?)
- 7. Does the series  $\sum_{n=1}^{\infty} \frac{(-1)^n 10^n}{n!}$  converge absolutely, converge conditionally, or diverge? (Why?)
- 7. Does the series  $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$  converge absolutely, converge conditionally, or diverge? (Why?)
- (a) Converges absolutely
- (b) Converges conditionally
- (c) Diverges

- **8.** For which values of p does the series  $\sum_{n=1}^{\infty} \frac{e^{np}}{1+n^2}$  converge?
- 8. For which values of p does the series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n^p}$  converge?
- 8. For which values of p does the series  $\sum_{n=1}^{\infty} \frac{n^p}{1+n^p}$  converge?
- (a) p < -1
- (b) p > 1

- (c) p > 4 (d)  $p \le 0$  (e) p < 0 (f) no value of p
- **9.** For which values of x does the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n^2 \, 8^n}$  converge?
- **9.** For which values of x does the series  $\sum_{n=1}^{\infty} \frac{n^2(x+2)^n}{(n+1)\,8^n}$  converge?
- **9.** For which values of x does the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n^3 \, 8^n}$  converge?
- (a) -10 < x < 6

(b)  $-10 \le x \le 6$ 

(c)  $-10 < x \le 6$ 

(d)  $-6 < x \le 10$ 

(e)  $-6 \le x < 10$ 

(f)  $-6 \le x \le 10$ 

- 10. The first few nonzero terms of the Taylor series for  $f(x) = \cos x$  centered at the point  $x = \frac{\pi}{4}$  are
- 10. The first few nonzero terms of the Taylor series for  $f(x) = \sin x$  centered at the point  $x = \frac{\pi}{4}$  are
- 10. The first few nonzero terms of the Taylor series for  $f(x) = \sqrt{x}$  centered at the point  $x = \frac{1}{2}$  are

(a) 
$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \left( x - \frac{\pi}{4} \right) + \frac{\sqrt{2}}{4} \left( x - \frac{\pi}{4} \right)^2 + \frac{\sqrt{2}}{4} \left( x - \frac{\pi}{4} \right)^3 + \cdots$$

(b) 
$$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \left( x - \frac{\pi}{4} \right) - \frac{\sqrt{2}}{4} \left( x - \frac{\pi}{4} \right)^2 + \frac{\sqrt{2}}{12} \left( x - \frac{\pi}{4} \right)^3 + \cdots$$

(c) 
$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \left( x - \frac{\pi}{4} \right) - \frac{\sqrt{2}}{4} \left( x - \frac{\pi}{4} \right)^2 - \frac{\sqrt{2}}{12} \left( x - \frac{\pi}{4} \right)^3 + \cdots$$

(d) 
$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \left(x - \frac{1}{2}\right) + \frac{\sqrt{2}}{4} \left(x - \frac{1}{2}\right)^2 - \frac{\sqrt{2}}{4} \left(x - \frac{1}{2}\right)^3 + \cdots$$

(e) 
$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \left( x - \frac{1}{2} \right) - \frac{\sqrt{2}}{4} \left( x - \frac{1}{2} \right)^2 + \frac{\sqrt{2}}{4} \left( x - \frac{1}{2} \right)^3 + \cdots$$

(f) 
$$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \left( x - \frac{1}{2} \right) + \frac{\sqrt{2}}{4} \left( x - \frac{1}{2} \right)^2 - \frac{\sqrt{2}}{12} \left( x - \frac{1}{2} \right)^3 + \cdots$$

- 11. Which of the following is closest to the value of  $\int_0^{0.5} e^{-x^2} dx$ ? Justify your answer by showing that the error is smaller than could possibly result in a neighboring answer.
- 11. Which of the following is closest to the value of  $\int_0^{0.5} \frac{\sin x}{x} dx$ ? Justify your answer by showing that the error is smaller than could possibly result in a neighboring answer.
- 11. Which of the following is closest to the value of  $\int_0^{0.5} \frac{\sin x}{x} dx$ ? Justify your answer by showing that the error is smaller than could possibly result in a neighboring answer.
- (a) 0.44
- (b) 0.45
- (c) 0.46
- (d) 0.47
- (e) 0.48
- (f) 0.49

Answers to first midterm (there were four versions of each problem, answers as follows):

- 1. A, B, E, F
- 2. B, D, C, E
- 3. D, C, E, F
- 4. C, F, B, A
- 5. F, C, D, E
- 6. B, C, D, E
- 7. A, C, E, F
- 8. A, B, C, D
- 9. B, C, D, F
- 10. C, D, E, F

Answers to second midterm (there were four versions of each problem, answers as follows):

- 1. C, D, E, B
- 2. A, F, D, E
- 3. C, D, A, E
- 4. A, B, E, D
- 5. B, C, D, F
- 6. A, C, E, F
- 7. D, E, C, F
- 8. A, C, E, F
- 9. A, B, D, F
- 10. B, D, A, C

Answers to third midterm (there were three versions of each problem, answers as follows):

- 1.
- (1.1)A, A, B
- (1.2) A, B, A

- (1.3) B, B, B
- (1.4) A, A, B
- 2. A, C, E
- 3. A, B, A
- 4. A, B, A
- 5. A, B, A
- 6. D, B, C
- 7. B, A, B
- 8. D, B, A
- 9. F, A, B
- 10. B, C, E
- 11. C, F, E