MATH 104 – Final Exam - Fall 2015

1. The first few nonzero terms of the Maclaurin series for $f(x) = \ln(1 + \sin x)$ are

(a) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{24}x^3 + \cdots$ (b) $1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{48}x^3 + \cdots$

(c) $x - \frac{1}{2}x^2 + \frac{1}{8}x^3 - \frac{1}{24}x^4 + \cdots$ (d) $1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{6}x^4 + \cdots$

(e) $x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \cdots$ (f) $1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{12}x^4 + \cdots$

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2. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n}(2n)!}$ If the series diverges, explain why.

(a) $\ln 2$ (b) $\ln 3 - \ln 2$ (c) $1/e^2$ (d) $1/2$ (e) $2/e$ (f) Diverges

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3. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n\, 4^n}$ converge?

(a) $-3 < x < 5$ (b) $-3 \le x < 5$ (c) $-3 < x \le 5$

(d) $-5 < x \le 3$ (e) $-5 \le x < 3$ (f) $-5 \le x \le 3$

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4. Determine whether the following series converge or diverge.

(1) $\sum_{n=1}^{\infty} \frac{n^3}{n^4 + 4}$ (2) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ (3) $\sum_{n=1}^{\infty} \frac{\ln(\ln(n))}{\ln n}$ (4) $\sum_{n=1}^{\infty} \frac{3n^2}{(n!)^2}$

(1) converge or diverge (2) converge or diverge

(3) converge or diverge (4) converge or diverge

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5. What is the limit of the sequence $\left\{ n^2 \left(1 - \cos \frac{1}{n} \right) \right\}$?

(a) 1 (b) -1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$ (e) $-\frac{\sqrt{3}}{2}$ (f) diverges

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6. The solution of the initial-value problem: $x\dfrac{dy}{dx} + 3y = 7x^4 \qquad y(1) = 1$

satisfies $y(2) =$

(a) 0 (b) 1 (c) 2 (d) 4 (e) 8 (f) 16

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7. The solution of the initial-value problem: $\dfrac{dy}{dx} - 20x^4e^{-y} = 0 \qquad y(0) = 0$

satisfies $y(1) =$

(a) $\ln 5$ (b) $\ln 4$ (c) $\ln 3$ (d) $\ln 2$ (e) 1 (f) 0

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8. The function

$$f(x) = \begin{cases} \dfrac{k}{x^3} & 1 < x \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for a certain value of k. For that probability density function, find the probability that $x > 2$

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{2}{3}$ (e) $\dfrac{1}{5}$ (f) $\dfrac{1}{6}$

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9. An object moves in such a way that its acceleration at time t seconds is $\dfrac{1}{t^2 + 5t + 6}$ meters per second^2. If the initial velocity of the object is $\ln \frac{2}{3}$ meters per second, what is the limit of its velocity as $t \to \infty$?

(a) $\ln \dfrac{3}{2}$ meters per second (b) $\ln 6$ meters per second (c) 1 meters per second

(d) $\ln \dfrac{4}{9}$ meters per second (e) $\ln \dfrac{9}{4}$ meters per second (f) 0 meters per second

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10. $\displaystyle\int_0^{\pi/8} \tan^4 2x \sec^4 2x \, dx$

(a) $\dfrac{4}{9}$ (b) $\dfrac{7}{24}$ (c) $\dfrac{5}{14}$ (d) $\dfrac{9}{28}$ (e) $\dfrac{6}{35}$ (f) $\dfrac{1}{7}$

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11. $\displaystyle\int_{\frac{1}{2}}^{\infty} \frac{\ln(2x)}{x^2} \, dx$

(a) $1 - \ln 2$ (b) 2 (c) $\ln 2 - \frac{1}{2}$ (d) $\frac{1}{2}$ (e) $2 - 2\ln 2$ (f) the integral diverges

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12. Find the y-coordinate of the centroid of the region in the first quadrant bounded by the coordinate axis and the graph of $y = \cos x$ for $0 \le x \le \frac{\pi}{2}$, if the density is constant.

(a) $\frac{\pi}{18}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{6}$ (e) $\frac{\pi}{4}$ (f) $\frac{\pi}{2}$

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13. Find the length of the part of the curve $y = \frac{3}{16}e^{2x} + \frac{1}{3}e^{-2x}$ for $0 \le x \le \ln 2$.

(a) $\frac{13}{16}$ (b) $\frac{11}{16}$ (c) $\frac{3}{8}$ (d) $\frac{9}{8}$ (e) $\frac{29}{64}$ (f) $\frac{3}{4}$

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14. The region between the graph of $y = 1 - x^2$ and the x-axis is rotated around the line $y = 1$. What is the volume of the resulting solid?

(a) $\frac{2\pi}{5}$ (b) $\frac{4\pi}{5}$ (c) $\frac{6\pi}{5}$ (d) $\frac{8\pi}{5}$ (e) 2π (f) $\frac{12\pi}{5}$

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15. Calculate the volume of the solid obtained by rotating the area between the graph of $y = \frac{1}{\sqrt{x^2 - 1}}$ and the x-axis for $1 < x < \sqrt{5}$ around the y-axis.

(a) π (b) 4π (c) 6π (d) 8π (e) 3π (f) 2π
