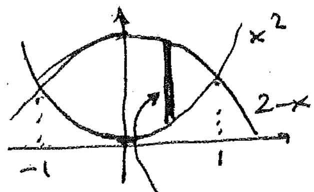
Sec 6.1 p373 #2




diameter of disk $= 2 - x^2 - x^2 = 2 - 2x^2$

radius of disk $= 1 - x^2$

area of disk $= \pi(1-x^2)^2$

$\Rightarrow$ Volume of one slice $= \pi(1-x^2)^2\,dx$

$\Rightarrow$ total volume $= \int_{-1}^{1} \pi(1-x^2)^2\,dx$

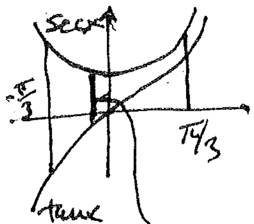
$= \pi \int_{-1}^{1} 1 - 2x^2 + x^4\,dx$

$= \pi\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\Big|_{-1}^{1}$

$= \pi\left(2 - \frac{4}{3} + \frac{2}{5}\right) = \underline{\frac{16\pi}{15}}$.

---

Sec 6.1 p373 #6




$\left.\begin{array}{l}\text{part (a): diameter of disk}\\ \text{part (b): side of square}\end{array}\right\} = \sec x - \tan x$

(a) radius of disk $= \frac{1}{2}(\sec x - \tan x)$

area of disk $= \frac{\pi}{4}(\sec x - \tan x)^2$

Volume of one slice $= \frac{\pi}{4}(\sec x - \tan x)^2\,dx$

Total volume $= \int_{-\pi/3}^{\pi/3} \frac{\pi}{4}(\sec^2 x - 2\sec x \tan x + \tan^2 x)\,dx$

$= \frac{\pi}{4}\int_{-\pi/3}^{\pi/3} \sec^2 x - 2\sec x \tan x + \sec^2 x - 1\,d$

$= \frac{\pi}{4}\int_{-\pi/3}^{\pi/3} 2\sec^2 x - 2\sec x \tan x - 1\,dx = \frac{\pi}{4}(2\tan x - 2\sec x -$

$= \frac{\pi}{4}\left(2(\sqrt{3} - (-\sqrt{3})) - 2\left(\frac{2}{\sqrt{3}} - \frac{2}{\sqrt{3}}\right) - \left(\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right)\right) = \pi\sqrt{3} - \frac{\pi^2}{6}$

(b) side of square $= \sec x - \tan x$

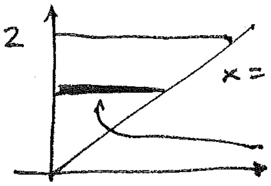
area of square $= (\sec x - \tan x)^2$

Vol of one slice $= (\sec x - \tan x)^2\,dx$

Total volume $= \int_{-\pi/3}^{\pi/3} (\sec x - \tan x)^2\,dx = 2\tan x - 2\sec x - x\Big|_{-\pi/3}^{\pi/3} = 4\sqrt{3} - \frac{2\pi}{3}$

---

Sec 6.1 p373 #16




radius of disk $= \frac{3y}{2}$.

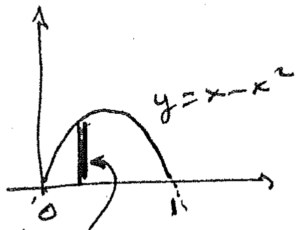
Volume of one disk $= \pi\left(\frac{3y}{2}\right)^2 dy$

Total volume $= \int_0^2 \pi \frac{9}{4} y^2\,dy = \frac{3}{4}\pi y^3 \Big|_0^2$

$= \underline{6\pi}$.

---

Sec 6.1 p373 #22




radius of disk $= x - x^2$

Volume of one disk $= \pi(x-x^2)^2 dx$

Total volume $= \int_0^1 \pi\,(x^2 - 2x^3 + x^4)\,dx$

$= \pi\left(\frac{1}{3}x^3 - \frac{1}{2}x^4 + \frac{1}{5}x^5\right)\Big|_0^1$

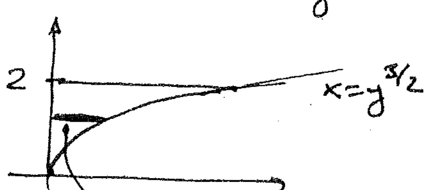
$= \pi\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = \frac{\pi}{30}$.

---

Sec 6.1 p373 #32

$x = y^{3/2}$ $\quad x = 0 \quad y = 2$

around y-axis




radius of disk $= y^{3/2}$

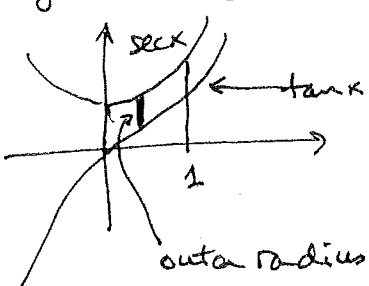
Volume of one disk $= \pi y^3 dy$

Total volume $= \int_0^2 \pi y^3 dy = \frac{\pi}{4} y^4 \Big|_0^2 = 4\pi$

---

Sec 6.1 p373 #44

$y = \sec x \quad y = \tan x \quad x = 0 \quad x = 1$ $\quad$ around x-axis




outer radius $= \sec x$

inner radius $= \tan x$

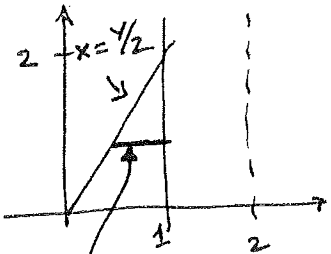
Volume of one washer $= \pi \sec^2 x\,dx - \pi \tan^2 x\,dx$

Total volume $= \int_0^1 \pi\,(\sec^2 x - \tan^2 x)\,dx$

$= \int_0^1 \pi\left(\sec^2 x - (\sec^2 x - 1)\right) dx$

$= \int_0^1 \pi\,dx = \underline{\underline{\pi}}$

Sec 6.1 p373 #52




around $x=1$, radius of disk is $1-y/2$

around $x=2$, outer radius is $2-y/2$
inner radius is 1

(a) Vol of one disk $= \pi\left(1-\frac{y}{2}\right)^2 dy$

$$\text{Total volume} = \int_0^2 \pi\left(1-y+\tfrac{1}{4}y^2\right)dy$$

$$= \pi\left(y-\tfrac{1}{2}y^2+\tfrac{1}{12}y^3\right)\Big|_0^2$$

$$= \pi\left(2-2+\tfrac{8}{12}\right) = \frac{2\pi}{3}.$$

(b) Vol of one washer:

$$\pi\left(2-\tfrac{y}{2}\right)^2dy - \pi(1)^2dy$$

$$\text{Total volume} = \int_0^2 \pi\left[\left(2-\tfrac{y}{2}\right)^2-1\right]dy$$

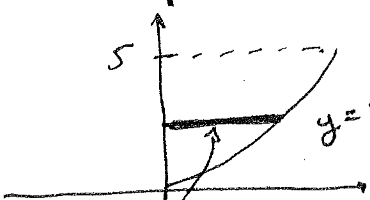
$$= \int_0^2 \pi\left(4-2y+\tfrac{1}{4}y^2-1\right)dy$$

$$= \pi\left(3y-y^2+\tfrac{1}{12}y^3\right)\Big|_0^2 = \pi\left(2+\tfrac{2}{3}\right)$$

$$= \frac{8\pi}{3}.$$

---

Sec 6.1 p373 #56




$y=x^2/2$ or $x=\sqrt{2y}$

radius of disk $=\sqrt{2y}$

Vol of one disk $= \pi\cdot 2y\,dy$

(a) Vol of bowl $= \int_0^5 \pi\cdot 2y\,dy = \pi y^2\Big|_0^5 = 25\pi$

(b) Vol of water up to height $h$:

$$V(h) = \int_0^h 2\pi y\,dy \Rightarrow \frac{dV}{dh} = 2\pi h.$$

$$\Rightarrow \frac{dV}{dt} = 2\pi h\frac{dh}{dt}.$$

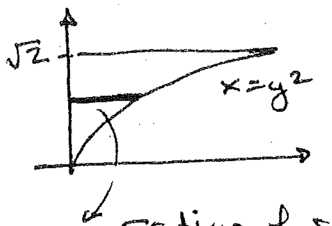
If $\frac{dV}{dt}=3$, then when $h=4$:

$$3 = 2\pi(4)\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{3}{8\pi}\ \frac{\text{units}}{\text{sec}}$$

---

Sec 6.2 p381 #3

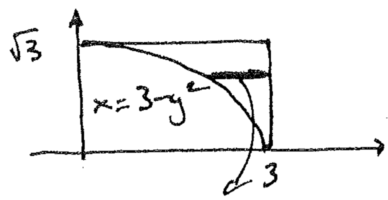



radius of shell $= y$
"height" of shell $= y^2$
Volume of one shell: $2\pi y(y^2)\,dy$

Total volume $= \int_0^{\sqrt{2}} 2\pi y^3\,dy = \frac{\pi}{2}y^4\Big|_0^{\sqrt{2}} = \underline{\underline{2\pi}}$

---

Sec 6.2 p381 #4

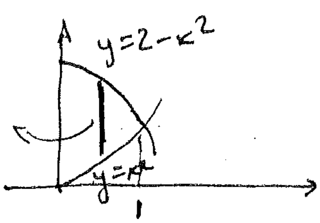



radius of shell $= y$
"height" of shell $= 3-(3-y^2) = y^2$
Volume of one shell $= 2\pi y(y^2)\,dy$

Total volume $= \int_0^{\sqrt{3}} 2\pi y^3\,dy = \frac{\pi}{2}y^4\Big|_0^{\sqrt{3}} = \underline{\underline{\frac{9\pi}{2}}}$

---

Sec 6.2 p381 #10



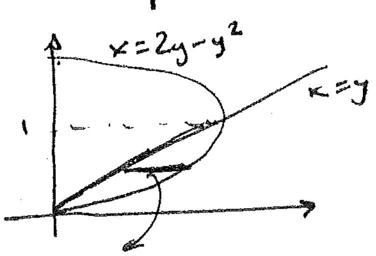

radius of shell $= x$
height of shell $= 2-x^2-x^2 = 2-2x^2$
Volume of one shell $= 2\pi x(2-2x^2)\,dx$

Total volume $= \int_0^1 2\pi(2x-2x^3)\,dx$

$= 2\pi\left(x^2-\frac{1}{2}x^4\right)\Big|_0^1 = \underline{\underline{\pi}}$

---

## Sec 6.2 p381 #18




radius of shell $= y$

"height" of shell $= 2y-y^2-y = y-y^2$

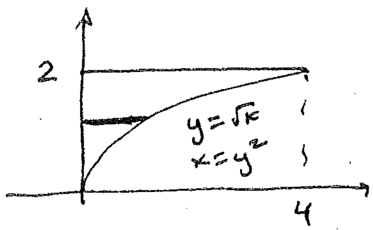
Volume of one shell $= 2\pi y(y-y^2)$

Total volume $= \int_0^1 2\pi(y^2-y^3)\,dy$

$= 2\pi\left(\frac{1}{3}y^3-\frac{1}{4}y^4\right)\Big|_0^1 = \frac{\pi}{6}.$

---

## Sec 6.2 p381 #32




(a) Around the x-axis: Shells.

radius of shell $= y$

"height" of shell $= y^2$

Volume of one shell $= 2\pi y(y^2)$

Total volume $= \int_0^2 2\pi y^3\,dy = \frac{\pi}{2}y^4\Big|_0^2 = 8\pi$

(b) Around the y-axis: Disks!

radius of disk $= y^2$

Volume of one disk: $\pi(y^2)^2dy = \pi y^4dy$

Total volume $= \int_0^2 \pi y^4\,dy = \frac{\pi}{5}y^5\Big|_0^2 = \frac{32\pi}{5}$

(c) Around $x=4$ – Washers

outer radius $= 4$

inner radius $= 4-y^2$

Vol. of one washer $= (\pi 4^2 - \pi(4-y^2)^2)dy$

Total volume $= \int_0^2 \pi\left(16-[16-8y^2+y^4]\right)dy$

$= \int_0^2 \pi(8y^2-y^4)\,dy = \pi\left(\frac{8}{3}y^3-\frac{1}{5}y^5\right)\Big|$

$= \pi\left(\frac{64}{3}-\frac{32}{5}\right) = 32\pi\left(\frac{10}{15}-\frac{3}{15}\right) = \frac{224\pi}{15}$

(d) Around $y=2$: Shells

radius of shell $= 2-y$

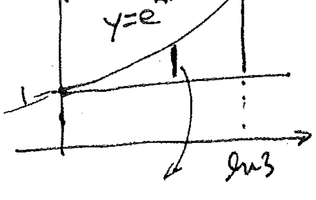
"height" of shell $= y^2$

Volume of one shell $= 2\pi(2-y)y^2dy$

Total volume $= \int_0^2 2\pi(2y^2-y^3)\,dy = 2\pi\left(\frac{2}{3}y^3-\frac{1}{4}y^4\right)\Big|_0^2 = 2\pi\left(\frac{16}{3}-\frac{16}{4}\right) = \frac{8\pi}{3}.$

---

## Sec 6.2 p381 #48




Washers: outer radius $= e^{x/2}$

inner radius $= 1$

Volume of one washer $= \pi\left((e^{x/2})^2-1^2\right)dx$

$= \pi(e^x-1)\,dx$

Total volume $= \int_0^{\ln 3} \pi(e^x-1)\,dx$

$= \pi(e^x-x)\Big|_0^{\ln 3}$

$= \pi(3-\ln 3-(1-0))$

$= \pi(2-\ln 3).$

Sec. 6.3 p388 # 6

Length of $x = \frac{y^3}{6} + \frac{1}{2y}$, $2 \le y \le 3$.

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{y^4}{4} - \frac{1}{2} + \frac{1}{4y^4}$$

$$1 + \left(\frac{dx}{dy}\right)^2 = \frac{y^4}{4} + \frac{1}{2} + \frac{1}{4y^4} = \left(\frac{y^2}{2} + \frac{1}{2y^2}\right)^2$$

$$\Rightarrow ds = \left(\frac{y^2}{2} + \frac{1}{2y^2}\right) dy$$

$$L = \int_2^3 \frac{y^2}{2} + \frac{1}{2y^2}\, dy$$

$$= \frac{y^3}{6} - \frac{1}{2y}\Big|_2^3$$

$$= \frac{9}{2} - \frac{1}{6} - \left(\frac{4}{3} - \frac{1}{4}\right)$$

$$= \frac{19}{4} - \frac{3}{2} = \underline{\underline{\frac{13}{4}}}.$$

---

Sec 6.3 p388 # 8

Length of $y = \frac{x^3}{3} + x^2 + x + \frac{1}{4x+4}$, $0 \le x \le 2$

$$\frac{dy}{dx} = \underbrace{x^2 + 2x + 1}_{(x+1)^2} - \frac{1}{4(x+1)^2}$$

$$\left(\frac{dy}{dx}\right)^2 = (x+1)^4 - \frac{1}{2} + \frac{1}{16(x+1)^4}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = (x+1)^4 + \frac{1}{2} + \frac{1}{16(x+1)^4} = \left((x+1)^2 + \frac{1}{4(x+1)^2}\right)^2$$

$$\Rightarrow ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \left((x+1)^2 + \frac{1}{4(x+1)^2}\right) dx$$

$$L = \int_0^2 (x+1)^2 + \frac{1}{4(x+1)^2}\, dx = \frac{1}{3}(x+1)^3 - \frac{1}{4(x+1)}\Big|_0^2$$

$$= 9 - \frac{1}{12} - \left(\frac{1}{3} - \frac{1}{4}\right) = 9 - \frac{1}{6} = \underline{\underline{\frac{53}{6}}}.$$

---

Sec 6.3 p388 #14

$$y = \int_{-2}^{x} \sqrt{3t^4 - 1}\, dt \qquad -2 \le x \le -1.$$

$$\frac{dy}{dx} = \sqrt{3x^4 - 1}$$

$$\left(\frac{dy}{dx}\right)^2 = 3x^4 - 1$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 3x^4$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{3}\, x^2\, dx$$

$$L = \int_{-2}^{-1} \sqrt{3}\, x^2\, dx = \frac{\sqrt{3}}{3} x^3 \Big|_{-2}^{-1} = \frac{\sqrt{3}}{3}(-1 - (-8))$$

$$= \frac{7\sqrt{3}}{3}.$$

---

Sec 6.3 p388 #26

$$y = (1 - x^{2/3})^{3/2} \qquad \frac{\sqrt{2}}{4} \le x \le 1.$$

$$\frac{dy}{dx} = \frac{3}{2}(1 - x^{2/3})^{1/2} \cdot \frac{2}{3} x^{-1/3} = \frac{(1 - x^{2/3})^{1/2}}{x^{1/3}}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{1 - x^{2/3}}{x^{2/3}} = x^{-2/3} - 1.$$

$$\left(\frac{dy}{dx}\right)^2 + 1 = x^{-2/3}$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= x^{-1/3}\, dx$$

$$L = \int_{\frac{\sqrt{2}}{4}}^{1} x^{-1/3}\, dx = \frac{3}{2} x^{2/3} \Big|_{\sqrt{2}/4}^{1}$$

$$= \frac{3}{2} - \frac{3}{2}\left(\frac{2}{16}\right)^{1/3}$$

$$= \frac{3}{2} - \frac{3}{4} = \frac{3}{4}.$$

So length of entire astroid $= 8L = \underline{\underline{6}}.$

---

Sec 6.3 p388 #36

$ds$ for the curve in #8 was $\left((x+1)^2 + \frac{1}{4(x+1)^2}\right) dx.$

So $L(x) = \int_0^x (t+1)^2 + \frac{1}{4(t+1)^2}\, dt = \frac{1}{3}(t+1)^3 - \frac{1}{4(t+1)} \Big|_0^x$

$$= \frac{1}{3}(x+1)^3 - \frac{1}{4(x+1)} - \left(\frac{1}{3} - \frac{1}{4}\right)$$

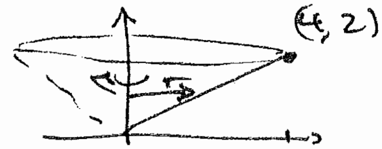
$$= \frac{1}{3}(x+1)^3 - \frac{1}{4(x+1)} - \frac{1}{12}$$

So $L(1) = \frac{1}{3}(8) - \frac{1}{4(2)} - \frac{1}{12} = \frac{8}{3} - \frac{1}{8} - \frac{1}{12} = \frac{64 - 3 - 2}{24}$

$$= \underline{\frac{59}{24}}.$$

Sec 6.4 p 393 # 10

$y = \frac{x}{2}$, $0 \le x \le 4$ around y-axis
$\downarrow$
radius $= x$.




$\frac{dy}{dx} = \frac{1}{2}$, $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}$ $\quad 1 + \left(\frac{dy}{dx}\right)^2 = \frac{5}{4}$

$ds = \frac{\sqrt{5}}{2} dx$. Surf area $= 2\pi \int_0^4 \frac{\sqrt{5}}{2} x \, dx = \frac{\pi\sqrt{5}}{2} x^2 \Big|_0^4$

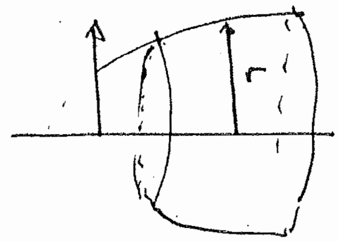
$= \underline{\underline{8\sqrt{5}\,\pi}}$

( Formula: $\frac{2\pi}{2} \cdot 4 \cdot \sqrt{16+4} = \pi 8\sqrt{5}$ ✓ )

---

Sec 6.4 p 393 # 16

$y = y = \sqrt{x+1}$ $\quad 1 \le x \le 5$ $\quad$ around x-axis




$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}}$, $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4(x+1)}$

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1}{4(x+1)}$

$r = y$
$\downarrow$

$ds = \sqrt{1 + \frac{1}{4(x+1)}}\, dx$. Surf area $= \int_1^5 2\pi \sqrt{x+1} \sqrt{1 + \frac{1}{4(x+1)}}\, dx$

$= \int_1^5 2\pi \sqrt{x + 1 + \frac{1}{4}}\, dx = \int_1^5 2\pi \sqrt{x + \frac{5}{4}}\, dx$

$= 2\pi \frac{2}{3}\left(x + \frac{5}{4}\right)^{3/2} \Big|_1^5 = \frac{2\pi}{4 \cdot 3} (4x+5)^{3/2} \Big|_1^5$

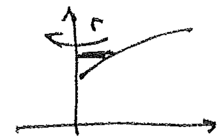
$= \frac{2\pi}{12}\left[25^{3/2} - 9^{3/2}\right] = \frac{2\pi}{12}[125 - 27] = \underline{\underline{\frac{196\pi}{12}}}$

$= \underline{\underline{\frac{49\pi}{3}}}$

---

Sec 6.4 p393 # 22

$y = \frac{1}{3}(x^2+2)^{3/2} \quad 0 \le x \le \sqrt{2}$, around y-axis



$y' = x(x^2+2)^{1/2}, \quad y'^2 = x^2(x^2+2)$

$\sqrt{1+y'^2} = \sqrt{1+x^2(x^2+2)}$.

$$S.A. = \int_0^{\sqrt{2}} 2\pi x \sqrt{1+x^2(x^2+2)}\,dx = \int_0^2 \pi \sqrt{1+u(u+2)}\,du$$

$u = x^2$
$du = 2x\,dx$

$$= \int_0^2 \pi\sqrt{1+2u+u^2}\,du$$

$$= \int_0^2 \pi\sqrt{(1+u)^2}\,du = \int_0^2 \pi(1+u)\,du$$

$$= \pi\left(u+\tfrac{1}{2}u^2\right)\Big|_0^2 = \pi(4) = 4\pi.$$

---

Sec 6.4 p393 #28

For this problem, we need to calculate the ~~arclength~~ surface area of the part of the sphere between $x=a$ and $x=a+h$ and show that it depends only on $h$ (and not on $a$).

$y = \sqrt{r^2-x^2}$

$y' = \dfrac{-x}{\sqrt{r^2-x^2}}, \quad y'^2 = \dfrac{x^2}{r^2-x^2}$

$1+y'^2 = 1+\dfrac{x^2}{r^2-x^2} = \dfrac{r^2}{r^2-x^2}$

$ds = \sqrt{1+y'^2}\,dx = \dfrac{r}{\sqrt{r^2-x^2}}\,dx.$

Surface area: $\displaystyle\int_a^{a+h} 2\pi y\,ds = \int_a^{a+h} 2\pi\sqrt{r^2-x^2}\cdot\frac{r}{\sqrt{r^2-x^2}}\,dx = 2\pi r\int_a^{a+h} dx$

$$= \underline{2\pi r h}.$$

So the surface area does not depend on $a$.

Sec 6.4 p393 #32

$y = (1-x^{2/3})^{3/2} \qquad 0 \le x \le 1$ around x-axis

From #22 in Sec 6.3, we know that $ds = x^{-1/3}dx$.

So surf area $= \int_0^1 2\pi y \, ds = \int_0^1 2\pi (1-x^{2/3})^{3/2} x^{-1/3} dx$

$u = x^{2/3}$

$du = \frac{2}{3} x^{-1/3} dx$

$= \int_0^1 2\pi (1-u)^{3/2} \cdot \frac{3}{2} du$

$= 3\pi \left(-\frac{2}{5}(1-u)^{5/2}\right)\Big|_0^1$

$= 6\pi/5. \Rightarrow$ Total S.A $= \frac{12\pi}{5}$

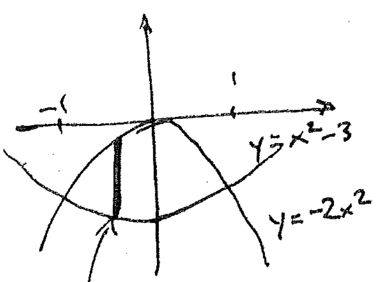
---

Sec 6.6 p413 #4

Center of mass, density $= \delta$, of region between $y = x^2 - 3$, $y = -2x^2$

Total mass $= \int_{-1}^1 \delta(-2x^2 - (x^2-3))dx = \int_{-1}^1 \delta(3-3x^2)dx$

$= \delta(3x - x^3)\Big|_{-1}^1 = 4\delta$




mass of this $= \delta(3-3x^2)dx$

$\bar{x} = x$

$\bar{y} = \frac{1}{2}(-2x^2 + (x^2-3)) = \frac{-x^2-3}{2}$

x-coord of center of mass

$= \frac{1}{4\delta} \int_{-1}^1 \delta(3-3x^2) x \, dx = \cdots = 0$ (obvious by symmetry)

y-coord of center of mass

$= \frac{1}{4\delta} \int_{-1}^1 \delta(3-3x^2)\left(\frac{-x^2-3}{2}\right)dx$

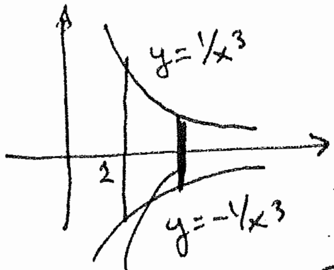
$= \frac{3}{8} \int_{-1}^1 (x^2-1)(x^2+3)dx = \frac{3}{8} \int_{-1}^1 x^4 + 2x^2 - 3 \, dx$

$= \frac{3}{8}\left(\frac{x^5}{5} + \frac{2x^3}{3} - 3x\right)\Big|_{-1}^1 =$

$= \frac{3}{8}\left(\frac{2}{5} + \frac{4}{3} - 6\right)$

$= \frac{1}{4}\left(\frac{3}{5} + 2 - 9\right) = -\frac{8}{5}.$

## Sec. 6.6 p413 #14




Total mass $= \int_1^a \delta \frac{2}{x^3}\,dx = -\delta \frac{1}{x^2}\Big|_1^a = \delta\left(1-\frac{1}{a^2}\right)$

mass $= \delta \frac{2}{x^3}dx$

$\bar{x} = x$

$\bar{y} = 0$

x-coord of center of mass

$= \frac{1}{\delta(1-\frac{1}{a^2})}\int_1^a \delta \frac{2}{x^3}\cdot x\,dx = \frac{a^2}{a^2-1}\int_1^a \frac{2}{x^2}\,dx$

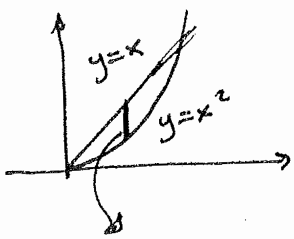
$= -\frac{2a^2}{a^2-1}\,\frac{1}{x}\Big|_1^a = \frac{2a^2}{a^2-1}\left(1-\frac{1}{a}\right)$

$= \frac{2a}{a+1}$ (So $\lim_{x\to\infty}\bar{x} = 2$).

y-coord of center of mass $= 0$. (obviously).

---

## Sec 6.6 p413 #16




Total mass $= \int_0^1 12x\cdot(x-x^2)\,dx = \int_0^1 12x^2-12x^3\,dx$

$= 4x^3-3x^4\Big|_0^1 = 1.$

mass $= 12x\cdot(x-x^2)dx$

$\bar{x} = x$

$\bar{y} = \frac{1}{2}(x+x^2)$

x-coord of center of mass

$= \frac{1}{1}\int_0^1(12x^2-12x^3)\cdot x\,dx = \int_0^1 12x^3-12x^4\,dx$

$= 3x^4-\frac{12}{5}x^5\Big|_0^1 = \frac{3}{5}.$

y-coord of center of mass

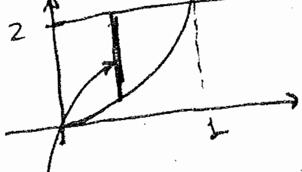
$= \frac{1}{1}\int_0^1(12x^2-12x^3)\cdot\frac{1}{2}(x+x^2)\,dx$

$= 6\int_0^1 x^3-x^5\,dx = 6\left(\frac{x^4}{4}-\frac{x^6}{6}\right)\Big|_0^1 = 6\cdot\frac{2}{24} = \frac{1}{2}$

---

## Sec 6.6 p413 #30

$y = x^2(x+1)$ $y=2,\ x=0$




mass $= (2-x^2(x+1))\,dx$

$\bar{x} = x$

$\bar{y} = \frac{1}{2}(2+x^2(x+1))$

Total mass $= \int_0^1 2-x^3-x^2\,dx = 2x-\frac{1}{4}x^4-\frac{1}{3}x^3\Big|_0^1 = \frac{24-7}{12} = \frac{17}{12}$

x-coord of centroid

$= \frac{12}{17}\int_0^1(2-x^3-x^2)x\,dx = \int_0^1\frac{12}{17}(2x-x^4-x^3)\,dx$

$= \frac{12}{17}\left(x^2-\frac{1}{5}x^5-\frac{1}{4}x^4\right)\Big|_0^1 = \frac{12}{17}\cdot\frac{11}{20} = \frac{33}{85}$

y-coord of centroid $= \frac{12}{17}\int_0^1\frac{1}{2}(4-x^4(x+1)^2)\,dx$

$= \frac{6}{17}\int_0^1 4-x^6-2x^5-x^4\,dx = \frac{6}{17}\left(4x-\frac{1}{7}x^7-\frac{1}{3}x^6-\frac{1}{5}x^5\right)\Big|_0^1$

$= \frac{6}{17}\left(\frac{420-71}{105}\right) = \frac{6}{17}\cdot\frac{349}{105} = \frac{698}{595}$

Math 104
Hand in Hw # 3

Name________________________________
Recitation Number_______________

**Spring 2014**

3. Find the arclength of the curve $y=\frac{2\sqrt{3}}{9}\left(3x^2+1\right)^{3/2}$ from $x=-1$ to $x=2$.

| | |
|---|---|
| (A) 8 | (E) 6 |
| (B) 2 | (F) 21 |
| (C) 9 | (G) 24 |
| (D) 4 | (H) 27 |

$y = \frac{2\sqrt{3}}{9}(3x^2+1)^{3/2}$

$y' = \frac{\sqrt{3}}{3}\cdot 6x(3x^2+1)^{1/2}$

$y'^2 = 12x^2(3x^2+1)$

$y'^2+1 = 36x^4+12x^2+1 = (6x^2+1)^2$

$ds = \sqrt{y'^2+1}\,dx = 6x^2+1\,dx$

$L = \int ds = \int_{-1}^{2} 6x^2+1\,dx$

$= 2x^3+x\Big|_{-1}^{2} = 18+3 = 21$

(F)

Math 104
Hand in Hw # 3

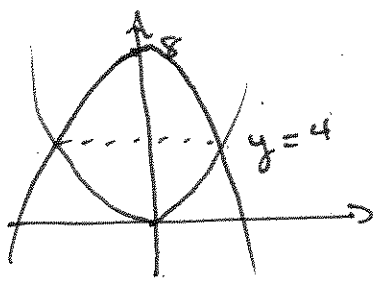
Name________________________________
Recitation Number_______________

**Spring 2013**

**PROBLEM 7:** What is the centroid of the region bounded by the curves $y = x^2$ and $y = 8 - x^2$?

Hint: draw a picture of this region as your first step.

(a) $(-2, 3)$ (b) $(2, 5)$ (c) $(-1, 4)$ (d) $(0, 4)$ (e) $(0, 3)$ (f) $(1, 4)$




The region is symmetric about the line $y=4$ (so $\bar{y} = 4$) and about the line $x=0$ (so $\bar{x} = 0$).

(D)

Math 104 Name________________________

Hand in Hw # 3 Recitation Number____________

**Fall 2012**

11. Suppose that the region bounded by $y = 4\tan(x^2)$ and the $x$-axis for $0 \le x \le \frac{\sqrt{\pi}}{2}$ is a thin homogeneous density plate of area $A$. Then the $x$-coordinate of the center of mass of the plate is:

(a) $\frac{2}{A}\pi^2$ (b) $\frac{2}{A}\pi$ (c) $\frac{1}{A}\ln 2$ (d) $\frac{3}{A}\sqrt{\pi}$ (e) $0$ (f) $\frac{e\pi}{2}$

Assume the density $\delta = 1$. $\Rightarrow$ $M = A$ and

$$M_y = \int_0^{\sqrt{\pi}/2} x \cdot 4\tan x^2 \, dx = \int_0^{\pi/4} 2\tan u \, du = \int_0^{\pi/4} 2\frac{\sin u}{\cos u} \, du \qquad \begin{pmatrix} v = \cos u \\ dv = -\sin u \, du \end{pmatrix}$$

$u = x^2$

$du = 2x\,dx$

$$= \int_1^{\sqrt{2}/2} -2\frac{dv}{v}$$

$$= -2\ln v \Big|_1^{\sqrt{2}/2} = -2\left[\ln\frac{\sqrt{2}}{2} - 0\right]$$

$$= -2\left[\tfrac{1}{2}\ln 2 - \ln 2\right] = \ln 2$$

$$\Rightarrow \bar{x} = \frac{M_y}{M} = \frac{\ln 2}{A} \quad (c)$$

Math 104 Name________________

Hand in Hw # 3 Recitation Number________

**Spring 2012**

12. What is the area of the surface obtained by rotating the part of the curve $y=\sqrt{4-x^2}$ from $x=0$ to $x=1$ around the $x-$axis?

A) $4\pi$ B) $2\pi$ C) $\pi$ D) $\sqrt{2}\pi$ E) $3\pi$ F) $8\pi$

$y=\sqrt{4-x^2}$

$y'=\dfrac{-x}{\sqrt{4-x^2}}$

$y'^2=\dfrac{x^2}{4-x^2}$

$1+y'^2=\dfrac{4-x^2}{4-x^2}+\dfrac{x^2}{4-x^2}=\dfrac{4}{4-x^2}$

$ds=\dfrac{2}{\sqrt{4-x^2}}\,dx$

$S.A. = \int 2\pi r\, ds$

around x-axis

$\Rightarrow r = y = \sqrt{4-x^2}$

$= \int_0^1 2\pi\sqrt{4-x^2}\,\dfrac{2}{\sqrt{4-x^2}}\,dx$

$= \int_0^1 4\pi\,dx = 4\pi$

A

Math 104 Name________________________
Hand in Hw # 3 Recitation Number____________

**Fall 2010**

7. What is the arclength of the part of the curve $y = \frac{1}{12}e^x + 3e^{-x}$ for $\ln 2 \leq x \leq \ln 4$?

(A) $\frac{5}{12}$ (B) $\frac{1}{2}$ (C) $\frac{7}{12}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$ (F) $\frac{5}{6}$ (G) $\frac{11}{12}$ (H) 1

---

$y = \frac{1}{12}e^x + 3e^{-x}$

$y' = \underbrace{\frac{1}{12}e^x} - \underbrace{3e^{-x}}$

$\frac{1}{12}e^x \cdot (-3e^{-x}) = -\frac{1}{4}$

$\Rightarrow$ IT'S ONE OF THOSE

$\Rightarrow L = \frac{1}{12}e^x - 3e^{-x} \Big|_{\ln 2}^{\ln 4}$

$= \frac{1}{12}(4-2) - 3\left(\frac{1}{4} - \frac{1}{2}\right)$

$= \frac{1}{6} + \frac{3}{4} = \frac{11}{12}$

G

Math 104 Name________________________________

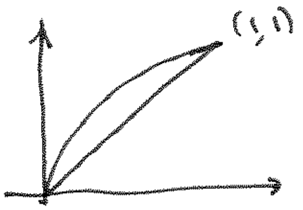
Hand in Hw # 3 Recitation Number______________

**Spring 2010**

10. An artist is designing a wine glass in a flower shape, which can be generated by rotating the region bounded by $y = \sqrt{x}$ and $x = y$, between $x = 0$ and $x = 1$, about $x$-axis. What is the surface area (which contains both the inside and the outside surfaces) of such a glass?

(a) $\left(\frac{8\sqrt{2}-4}{3} + \sqrt{2}\right)\pi$ (b) $\left(\frac{8\sqrt{2}-4}{3} + \sqrt{5}\right)\pi$ (c) $\left(\frac{8\sqrt{2}-4}{3} + 1\right)\pi$

(d) $\left(\frac{5\sqrt{5}-1}{6} + \sqrt{2}\right)\pi$ (e) $\left(\frac{5\sqrt{5}-1}{6} + \sqrt{5}\right)\pi$ (f) $\left(\frac{5\sqrt{5}-1}{6} + 1\right)\pi$




OUTSIDE SURFACE:

$y = \sqrt{x}$ $\qquad r = y = \sqrt{x}$ (around x-axis)

$y' = \frac{1}{2\sqrt{x}}$ $\qquad SA = \int 2\pi r\,ds = 2\pi \int_0^1 \sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx$

$y'^2 = \frac{1}{4x}$ $\qquad = 2\pi \int_0^1 \sqrt{x+\frac{1}{4}}\,dx$

$ds = \sqrt{1+\frac{1}{4x}}\,dx$

$$= 2\pi\,\frac{2}{3}\left(x+\frac{1}{4}\right)^{3/2}\Big|_0^1$$

$$= \frac{4\pi}{3}\left(\left(\frac{5}{4}\right)^{3/2} - \left(\frac{1}{4}\right)^{3/2}\right)$$

$$= \frac{4\pi}{3}\left(\frac{5\sqrt{5}}{8} - \frac{1}{8}\right) = \pi\left(\frac{5\sqrt{5}-1}{6}\right)$$

INSIDE SURFACE:

$y = x$ $\qquad r = y = x$

$y' = 1$ $\qquad SA = \int_0^1 2\pi x\sqrt{2}\,dx = \pi\sqrt{2}\,x^2\Big|_0^1 = \pi\sqrt{2}$

$y'^2 = 1$

$ds = \sqrt{2}\,dx$

TOTAL S.A. $= \pi\left[\frac{5\sqrt{5}-1}{6} + \sqrt{2}\right]$ D