Sec 7.2 #10

$$\frac{dy}{dx} = x^2\sqrt{y}$$

$$\int \frac{dy}{\sqrt{y}} = \int x^2\,dx$$

$$2\sqrt{y} = \frac{1}{3}x^3 + C$$

$$\underline{y = \frac{1}{4}\left(\frac{1}{3}x^3 + C\right)^2}$$

Check:

$$y' = \frac{1}{4}\,2\left(\frac{1}{3}x^3 + C\right)x^2 = \left[\frac{1}{2}\left(\frac{1}{3}x^2 + C\right)\right]x^2 = \sqrt{y}\cdot x^2 \checkmark$$

Sec 7.2 # 14

$$\sqrt{2xy}\,\frac{dy}{dx} = 1$$

$$\int \sqrt{y}\,dy = \int \frac{dx}{\sqrt{2x}}$$

$$\frac{2}{3}y^{3/2} = \sqrt{2x} + C$$

$$y = \left(\frac{3}{2}\left(\sqrt{2x} + C\right)\right)^{2/3}$$

Check:

$$y' = \left(\sqrt{2x} + C\right)^{-1/3}\cdot\frac{1}{\sqrt{2x}}\cdot\left(\frac{2}{3}\right)^{1/3}$$

$$\sqrt{2xy}\,\frac{dy}{dx} = \left(\frac{3}{2}\right)^{1/3}\left(\sqrt{2x} + C\right)^{1/3}\left(\sqrt{2x} + C\right)^{-1/3}\left(\frac{2}{3}\right)^{1/3} = 1 \checkmark$$

Sec 7.2 # 20

$$\frac{dy}{dx} = xy + 3x - 2y - 6$$

$$\frac{dy}{dx} = (x-2)(y+3)$$

$$\int \frac{dy}{y+3} = \int (x-2)\,dx$$

$$\ln(y+3) = \frac{1}{2}x^2 - 2x + C \quad \ln C$$

$$y + 3 = Ce^{\frac{1}{2}x^2 - 2x}$$

$$\to\ y = Ce^{\frac{1}{2}x^2 - 2x}\ \ 3$$

Check: $\dfrac{dy}{dx} = C(x-2)e^{\frac{1}{2}x^2 - 2x}$

$$xy + 3x - 2y - 6 = Cxe^{\frac{1}{2}x^2-2x} - 3x + 3x - 2Ce^{\frac{1}{2}x^2-2x} + 6 - 6$$

$$= C(x-2)e^{\frac{1}{2}x^2-2x} \checkmark$$

Sec 7.2 # 24 a)

$$\left.\begin{array}{l}\frac{dp}{dh} = kp \\ p(0) = p_0\end{array}\right\}\ p = p_0e^{kh}$$

Since $p(0) = 1013$, $p_0 = 1013$

$p(20) = 90 \Rightarrow 90 = 1013\,e^{20k}$

$$\underline{k = \frac{1}{20}\ln\frac{90}{1013} \approx -0.121}$$

b) For $h = 50$, $p = 1013\,e^{\frac{5}{2}\ln\frac{90}{1013}} \approx 2.383$ millibars

c) For $p = 900$, $900 = 1013\,e^{\frac{h}{20}\ln\frac{90}{1013}} \Rightarrow \frac{900}{1013} = e^{\frac{h}{20}\ln\frac{90}{1013}} \Rightarrow \ln\frac{900}{1013} = \frac{h}{20}\ln\frac{90}{1013}$

$$\Rightarrow h = \frac{20\ln\frac{900}{1013}}{\ln\frac{90}{1013}} \approx 0.977\ \text{km}$$

Sec 7.2 #30. Population growing exponentially.

t	P
0	?
3	10000
5	40000

$+2$, $\times 4$

In 2 hours population quadruples
$\Rightarrow$ in 1 hour, population doubles.
Or 1 hour ago, population was half

$\Rightarrow$

t	P
0	1250
1	2500
2	5000
3	10000

$\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$

$\Rightarrow$ Started with 1250 bacteria at $t=0$

Sec 7.2 #36. a) $\frac{dp}{dx} = -\frac{1}{100} p \Rightarrow P = P_0 e^{-\frac{1}{100}x}$, $P_0 = 20.09\, e$

$p(100) = 20.09 \Rightarrow 20.09 = P_0 e^{-1}$, $\underline{\underline{P = 20.09\, e^{1-\frac{1}{100}x}}}$

b) $p(10) = 20.09\, e^{9/10} \approx \$49.41$
$p(90) = 20.09\, e^{1/10} \approx \$22.20$

c) $r(x) = x\,p(x)$
$\frac{dr}{dx} = p(x) + x\frac{dp}{dx} = p(x) - \frac{x}{100}p = (1-\frac{x}{100})p$.
this is positive if $x<100$, zero if $x=100$ and negative if $x>100$
$\Rightarrow x=100$ is the maximum for $r(x)$.
(and $r(100) = \$2009$ is the maximum value).

d) 

Sec 7.2 #38 Half is left after 139 days. When is 0.05 (5%) left?

well:

$\frac{1}{2} = 0.5$	139 days
$\frac{1}{4} = 0.25$	278
$\frac{1}{8} = 0.125$	417
$\frac{1}{16} = 0.0625$	556
$\frac{1}{32} = 0.03125$	695

So sometime between 556 & 695 days (closer to 556).

Officially: $A = A_0 e^{-kt}$ and $\frac{1}{2} = e^{-k(139)}$ so $k = \frac{\ln 2}{139}$

Want $0.05 = e^{-\frac{\ln 2}{139}t} \Rightarrow t = \ln 20 \cdot \frac{139}{\ln 2} \approx \underline{\underline{600.75}}$ days

Sec 7.2 #42 temp of beam: $u(t)$

$65-u(t)$ decays exponentially:

t (min)	$u(t)$	$65-u(t)$
0	?	?
10	35	30
20	50	15

$65-u(t)$ divides in half every 10 min, so at $t=0$ it was $60°$.

$\Rightarrow u(0) = 65-60 = 5°F$

Sec 9.1 #1 $y' = x+y \rightarrow y'=0$ where $y=-x$, $y'>0$ above that line, $y'<0$ below that line

Choice (D) does that.

#2 $y' = y+1 \rightarrow y'=0$ when $y=-1$ — all along horizontal line.
Only choice (C) does that

#3 $y' = -\frac{x}{y}$ slope is infinite when $y=0$ (on x-axis) and zero when $x=0$ (on y-axis).
Only choice (A) does that

#4 $y' = y^2-x^2$ — slope is zero when $y=x$ or $y=-x$
Only choice (B) does that

Sec 9.1 #10 $y = 1 + \int_0^x y(t)\,dt$ $\quad \frac{d}{dx}$ of both sides $\Rightarrow$ $\underline{y'=y}$. and $y(0)=1$

(so $y=e^x$)

Sec 9.1 #12 $y' = x(1-y)$ $\quad y(1)=0$ $\quad dx=0.2$

x	y	y'	$y+0.2y'$
1	0	1	0.2000
1.2	0.2	0.96	0.3920
1.4	0.392	0.8512	0.5622

Sec 9.1 #20 $y' = x\sin y$ $\quad y(0)=1$

x	y	y'	next $y = y + \frac{1}{3}y'$
0	1	0	1
1/3	1	0.2805	1.0935
2/3	1.0935	0.5922	1.2909
1	1.2909	0.9611	1.6112
4/3	1.6112	1.3322	2.0553
5/3	2.0553	1.4748	2.5469
2	2.5469	—	—

Exact sol:

$$\frac{dy}{\sin y} = x\,dx$$

$$-\ln(\csc y + \cot y) = \tfrac{1}{2}x^2 + \ln C$$

$$\csc y + \cot y = Ce^{-\frac{1}{2}x^2}$$

$$\frac{1+\cos y}{\sin y} = Ce^{-\frac{1}{2}x^2}$$

$$\cot\left(\frac{y}{2}\right) = Ce^{-\frac{1}{2}x^2}$$

Now: $y(0)=1$
$\Rightarrow C = \cot\frac{1}{2}$

$$y = 2\,\mathrm{arccot}\left(\left(\cot\tfrac{1}{2}\right)e^{-\frac{1}{2}x^2}\right)$$

$$y(2) = 2\,\mathrm{arccot}\left(\frac{\cot\frac{1}{2}}{e^2}\right)$$

$\approx 2.6559.$

Sec 9.2 #4 $\quad y' + \underbrace{(\tan x)}_{p} y = \underbrace{\cos^2 x}_{q}$ $\qquad y = e^{-\int p}\int e^{\int p} q$

$\int p = \int \tan x\,dx = -\ln\cos x$

$$= e^{\ln\cos x}\int e^{-\ln\cos x}\cos^2 x\,dx$$

$$= \cos x\int \cos x\,dx = \cos x(\sin x + C)$$

$$= \underline{\cos x\sin x + C\cos x}$$

Check: $y = \cos x\sin x + C\cos x$

$y' = \cos^2 x - \sin^2 x - C\sin x$

$y' + (\tan x)y = \cos^2 x - \sin^2 x - C\sin x$

$\qquad + \sin^2 x + C\sin x = \cos^2 x$ ✓

Sec 9.2 # 10 $\quad x\dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$ $\qquad y = e^{-\int p}\int e^{\int p} q = e^{-2\ln x}\int e^{2\ln x}\dfrac{\cos x}{x^2}\,dx$

$$\frac{dy}{dx} + \underbrace{\frac{2}{x}}_{p} y = \underbrace{\frac{\cos x}{x^2}}_{q}$$

$$= \frac{1}{x^2}\int x^2\frac{\cos x}{x^2}\,dx = \frac{1}{x^2}\int \cos x\,dx$$

$\int p = \int \frac{2}{x}\,dx = 2\ln x$

$$= \frac{1}{x^2}(\sin x + C) = \underline{\frac{\sin x}{x^2} + \frac{C}{x^2}}$$

Check: $y = \dfrac{\sin x}{x^2} + \dfrac{C}{x^2}$

$y' = \dfrac{\cos x}{x^2} - \dfrac{2\sin x}{x^3} - \dfrac{2C}{x^3}$

$xy' = \dfrac{\cos x}{x} - \dfrac{2\sin x}{x^2} - \dfrac{2C}{x^2}$

$\dfrac{\cos x}{x} - 2y = \dfrac{\cos x}{x} - \left[\dfrac{2\sin x}{x^2} + \dfrac{2C}{x^2}\right]$ ✓

Sec 9.2 #16. $\quad t\dfrac{dy}{dt} + 2y = t^3 \quad y(2) = 1$ $\qquad y = e^{-\int p}\int e^{\int p} q$

$$\frac{dy}{dt} + \underbrace{\frac{2}{t}}_{p} y = \underbrace{t^2}_{q}$$

$$= e^{-2\ln t}\int e^{2\ln t}\cdot t^2\,dt$$

$$= \frac{1}{t^2}\int t^4\,dt = \frac{1}{t^2}\left(\frac{1}{5}t^5 + C\right)$$

$\int p = \int \frac{2}{t}\,dt = 2\ln t$

$$= \frac{1}{5}t^3 + \frac{C}{t^2}.$$

Check: $y = \frac{1}{5}t^3 + \dfrac{C}{t^2}$

$y' = \frac{3}{5}t^2 - \dfrac{2C}{t^3}$

$ty' + 2y = \frac{3}{5}t^3 - \dfrac{2C}{t^2} + \frac{2}{5}t^3 + \dfrac{2C}{t^2} = t^3$ ✓

Sec 9.2 #26 6

a) $L\frac{di}{dt} + Ri = 0$ ie $\frac{di}{dt} = -\frac{R}{L} i \Rightarrow i(t) = i_0 e^{-\frac{R}{L}t}$

b) Half the original value when $e^{-\frac{R}{L}t} = \frac{1}{2}$ ie $\frac{R}{L}t = \ln 2 \Rightarrow t = \underline{\frac{L}{R}\ln 2}$.

c) $t = \frac{L}{R}$: $i(\frac{L}{R}) = i_0 e^{-\frac{R}{L}\cdot\frac{L}{R}} = \frac{i_0}{e}$.

Sec 9.3 #6 5

Orig curves: $y = cx^2$

$y' = 2cx$

divide to eliminate c:

$\frac{y'}{y} = \frac{2}{x}$

D.E. for original curves $y' = \frac{2y}{x}$

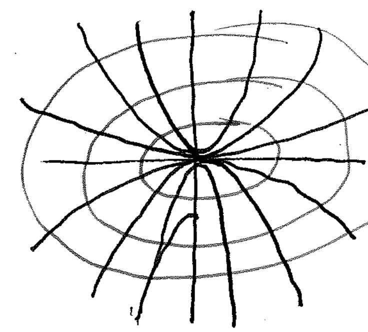
D.E. for orthog traj: $y' = -\frac{x}{2y}$

$2yy' = -x$

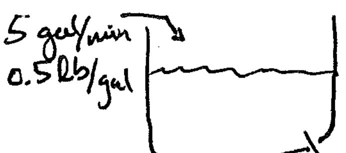
$2y\,dy = -x\,dx$

$y^2 = -\frac{1}{2}x^2 + C$

$\Rightarrow y^2 + \frac{1}{2}x^2 = C$ (ellipses)



Sec 9.3 #14



a) Since 5 gal/min in, 3 gal/min out, increase by 2 gal/min.
Initially 100 gal in 200 gal tank
$\Rightarrow$ Tank full in $\underline{\underline{50 \text{ min}}}$

b) A(t) = amount of concentrate in tank (lb) at time t (min).

$\frac{dA}{dt} = 5 \cdot 0.5 - \frac{A}{100+2t} \cdot 3$

$\frac{dA}{dt} = 2.5 - \frac{3A}{100+2t}$

$\frac{dA}{dt} + \underbrace{\frac{3}{100+2t}}_{P} A = \underbrace{2.5}_{q}$

$A = e^{-\int p} \int e^{\int p} q$. $\int p = \int \frac{3}{100+2t}\,dt$

$= \frac{3}{2}\ln(100+2t)$

$A = e^{-\frac{3}{2}\ln(100+2t)} \int 2.5\, e^{\frac{3}{2}\ln(100+2t)}\,dt$

$= (100+2t)^{-3/2} \int 2.5\,(100+2t)^{3/2}\,dt$

$= (100+2t)^{-3/2}\left[\frac{1}{2}(100+2t)^{5/2} + C\right]$.

$A(t) = 50 + t + C(100+2t)^{-3/2}$

$A(0) = 0 \Rightarrow 0 = 50 + C \cdot 100^{-3/2} = 50 + \frac{C}{1000} \Rightarrow C = -50000$

$A(t) = 50 + t - \frac{50000}{(100+2t)^{+3/2}}$.

$A(50) = 100 - \frac{50000}{(100+100)^{3/2}} = 100 - \frac{50000}{2\sqrt{2}\,1000}$

$= 100 - \frac{25}{\sqrt{2}}$ (lb) ≈ 82.3 lb

Sec 9.3 #16

$$\frac{0.3\,ft^3}{min}\times 0.04 \longrightarrow \boxed{} \longrightarrow 0.3\,\frac{ft^3}{min}$$

$V(t)$ = volume (ft^3) of CO in room at time t (min)

$$\frac{dV}{dt} = \text{Rate in} - \text{Rate out} = 0.3\times 0.04 - 0.3\times\frac{V}{4500}, \quad V(0)=0.$$

$$\frac{dV}{dt} = \frac{12}{1000} - \frac{V}{15000} \quad \text{ie} \quad \frac{dV}{dt} + \underbrace{\frac{1}{15000}}_{P} V = \underbrace{\frac{12}{1000}}_{Q}.$$

$$\int P = \frac{1}{15000}t \Rightarrow V = \frac{1}{e^{t/15000}} \int \frac{12}{1000} e^{t/15000}\,dt$$

$$= e^{-t/15000}\left[12\cdot 15\, e^{t/15000} + C\right]$$

$$= e^{-t/15000}\left[180\, e^{t/15000} + C\right] \qquad V(0)=0 \Rightarrow C=-180.$$

So $\underline{V = 180 - 180\,e^{-t/15000}}$

When concentration is $0.01\% = 0.0001$, amount (vol) of CO is $V = 0.45\,ft^3$.

So $0.45 = 180(1-e^{-t/15000})$

$0.0025 = 1 - e^{-t/15000}$

$e^{-t/15000} = 0.9975$

$\Rightarrow t = -15000\ln 0.9975 \approx \underline{37.55 \text{ min}}$