Sec 8.5 #10

$$\int \frac{dx}{x^2+2x} = \int \frac{1}{x(x+2)}dx = \int \frac{1/2}{x} - \frac{1/2}{x+2}dx = \frac{1}{2}\left(\ln x - \ln(x+2)\right) + C$$

$$= \frac{1}{2}\ln\left|\frac{x}{x+2}\right| + C$$

Sec 8.5 #16

$$\int \frac{x+3}{2x^3-8x}dx = \int \frac{x+3}{2x(x-2)(x+2)}dx = \int \frac{-\frac{3}{8}}{x} + \frac{\frac{5}{16}}{x-2} + \frac{\frac{1}{16}}{x+2}dx$$

$$= \frac{5}{16}\ln|x-2| + \frac{1}{16}\ln|x+2| - \frac{3}{8}\ln|x| + C.$$

Sec 8.5 #18

$$\int_{-1}^{0} \frac{x^3}{\underbrace{x^2-2x+1}_{(x-1)^2}} = \int_{-2}^{-1} \frac{(u+1)^3}{u^2} = \int_{-2}^{-1} u + 3 + \frac{3}{u} + \frac{1}{u^2}du = \frac{1}{2}u^2 + 3u + 3\ln|u| - \frac{1}{u}\Big|_{-2}^{-1}$$

$u = x-1$

$du = dx$

$(x = u+1)$

$$= \frac{1}{2}(1-4) + 3 - 3\ln 2 + 1 - \frac{1}{2} = \underline{\underline{2 - 3\ln 2}}$$

$\boxed{\text{OR}}$

$$= \int_{-1}^{0} x + 2 + \frac{3}{x-1} + \frac{1}{(x-1)^2}dx = \frac{1}{2}x + 2x + 3\ln|x-1| - \frac{1}{x-1}\Big|_{-1}^{0}$$

$$= -\frac{1}{2} - 2 - 3\ln 2 + 1 - \frac{1}{2} = \underline{\underline{2 - 3\ln 2}}$$

Sec 8.5 #22

$$\int_{1}^{\sqrt{3}} \frac{3t^2+t+4}{t(t^2+1)}dt = \int_{1}^{\sqrt{3}} \frac{4}{t} + \frac{At+B}{t^2+1}dt = \int_{1}^{\sqrt{3}} \frac{4}{t} - \frac{t}{t^2+1} + \frac{1}{t^2+1}dt$$

$3t^2+t+4 = 4t^2+4 + At^2+Bt$

$-t^2+t = At^2+Bt$

$\Rightarrow A=-1,\ B=1$

$$= 4\ln t - \frac{1}{2}\ln(t^2+1) + \arctan t\Big|_{1}^{\sqrt{3}}$$

$$= 2\ln 3 - \ln 2 + \frac{1}{2}\ln 2 + \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \underline{2\ln 3 - \frac{1}{2}\ln 2 + \frac{\pi}{12}}$$

Sec 8.5 #44

$$\int \frac{(x+1)^2\arctan(3x)+9x^3+x}{(9x^2+1)(x+1)^2}\,dx = \int \frac{\arctan(3x)}{9x^2+1}\,dx + \int \frac{x}{(x+1)^2}\,dx$$

$\left[u = \arctan 3x,\ du = \frac{3}{9x^2+1}\,dx\right]$ $\left[u = x+1,\ x = u-1,\ dx = du\right]$ $\frac{u-1}{u^2} = \frac{1}{u} - \frac{1}{u^2}$

$$= \frac{1}{6}(\arctan 3x)^2 + \ln|x+1| + \frac{1}{x+1} + C.$$

Sec 8.5 #48

$$\int \frac{1}{x\sqrt{x+9}}\,dx = \int \frac{2u}{(u^2-9)u}\,du = \int \frac{2}{(u-3)(u+3)}\,du = \int \frac{1/3}{u-3} + \frac{-1/3}{u+3}\,du$$

$u = \sqrt{x+9}$
$x = u^2 - 9$
$dx = 2u\,du$

$$= \frac{1}{3}\left(\ln|u-3| - \ln|u+3|\right) + C$$

$$= \frac{1}{3}\left(\ln\left|\sqrt{x+9}-3\right| - \ln\left|\sqrt{x+9}+3\right|\right) + C$$

$$= \frac{1}{3}\ln\left|\frac{\sqrt{x+9}-3}{\sqrt{x+9}+3}\right| + C$$

Sec 8.5 #56

$y = \frac{2}{(x+1)(2-x)}$ $x=1$, $x=0$, $y=0$ around x-axis

Shells: $V = \int 2\pi x y\,dx.$

$$V = 2\pi\int_0^1 \frac{2x}{(x+1)(2-x)}\,dx = 2\pi\int_0^1 \frac{-2/3}{x+1} + \frac{4/3}{2-x}\,dx = \frac{2\pi}{3}\left(-2\ln|x+1| - 4\ln|2-x|\right)\Big|_0^1$$

$$= \frac{2\pi}{3}\left(-2\ln 2 + 4\ln 2\right) = \frac{4\pi}{3}\ln 2.$$

Sec 8.6 #6

$$\int x(7x+5)^{3/2}\,dx = \int \frac{1}{7}(u-5)\,u^{3/2}\,\frac{1}{7}\,du = \frac{1}{49}\int u^{5/2} - 5u^{3/2}\,du = \frac{1}{49}\left(\frac{2}{7}u^{7/2} - 2u^{5/2}\right) + C$$

$u = 7x+5$
$x = \frac{1}{7}(u-5)$
$dx = \frac{1}{7}\,du$

$$= \frac{2}{49}u^{5/2}\left(\frac{1}{7}u - 1\right) + C = \frac{2}{49}(7x+5)^{5/2}\left(\frac{1}{7}(7x+5) - 1\right) + C$$

$$= \frac{2}{49}(7x+5)^{5/2}\left(x - \frac{2}{7}\right) + C.$$

Sec 8.6 #14

$$\int \frac{\sqrt{x^2-4}}{x}\,dx = \int \frac{2\tan\theta}{2\sec\theta}\, 2\sec\theta\tan\theta\, d\theta = \int 2\tan^2\theta\, d\theta$$

$$= \int 2\sec^2\theta - 2\, d\theta$$

$$= 2\tan\theta - 2\theta + C$$

$$= \sqrt{x^2-4} - 2\,\mathrm{arcsec}\,\frac{x}{2} + C$$

(or, better:)

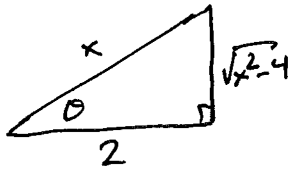
$$= \sqrt{x^2-4} - 2\arctan\frac{\sqrt{x^2-4}}{2} + C.$$

$x = 2\sec\theta \quad dx = 2\sec\theta\tan\theta\, d\theta$

$x^2-4 = 4\sec^2\theta - 4$

$= 4\tan^2\theta$

$\sec\theta = x/2$



Sec 8.6 #20

$$\int \frac{\arctan x}{x^2}\,dx = -\frac{\arctan x}{x} + \int \frac{dx}{x(1+x^2)} = -\frac{\arctan x}{x} \int \frac{1}{x} + \frac{-x}{1+x^2}\,dx$$

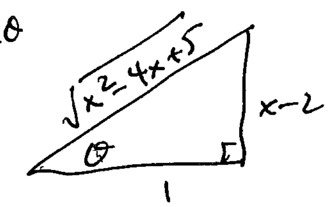
Parts: $u = \arctan x \rightarrow du = \frac{dx}{1+x^2}$

$dv = \frac{1}{x^2}dx \rightarrow v = -\frac{1}{x}$

$$= -\frac{\arctan x}{x} + \ln|x| - \frac{1}{2}\ln(1+x^2) + C.$$

Sec 8.6 #38

$$\int \frac{x^2}{\sqrt{x^2-4x+5}}\,dx = \int \frac{x^2}{\sqrt{(x-2)^2+1}}\,dx = \int \frac{(\tan\theta+2)^2\sec^2\theta\, d\theta}{\sec\theta}$$



$\tan\theta = x-2$

$dx = \sec^2\theta\, d\theta$

$x = \tan\theta + 2$

$$= \int (\underbrace{\tan^2\theta}_{\sec^2\theta - 1} + 4\tan\theta + 4)\sec\theta\, d\theta$$

$$= \int \underline{\underline{\sec^3\theta}} + 4\tan\theta\sec\theta + 3\sec\theta\, d\theta$$

$$\int \sec^3\theta\, d\theta = \sec\theta\tan\theta - \int \sec\theta\underbrace{\tan^2\theta}_{\sec^2\theta-1}\, d\theta$$

Parts

$u = \sec\theta \rightarrow du = \sec\theta\tan\theta\, d\theta$

$dv = \sec^2\theta\, d\theta \quad v = \tan\theta$

$$= \sec\theta\tan\theta + \int \sec\theta\, d\theta - \int \sec^3\theta\, d\theta$$

ie.

$$\int \sec^3\theta\, d\theta = \sec\theta\tan\theta + \ln(\sec\theta+\tan\theta) - \int \sec^3\theta\, d\theta$$

add, divide by 2:

$$\int \sec^3\theta\, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln(\sec\theta+\tan\theta) + C$$

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln(\sec\theta+\tan\theta) + 4\sec\theta + 3\ln(\sec\theta+\tan\theta) + C$$

$$= \frac{1}{2}\sec\theta\tan\theta + 4\sec\theta + \frac{7}{2}\ln(\sec\theta+\tan\theta) + C$$

$$= \frac{1}{2}(x-2)\sqrt{x^2-4x+5} + 4\sqrt{x^2-4x+5} + \frac{7}{2}\ln\left(x-2+\sqrt{x^2-4x+5}\right) + C$$

$$= \frac{1}{2}x\sqrt{x^2-4x+5} + 3\sqrt{x^2-4x+5} + \frac{7}{2}\ln\left|x-2+\sqrt{x^2-4x+5}\right| + C$$

Sec 8.7 #2 $\int_1^3 (2x-1)\,dx$ Trap. rule w/ $n=4$. $\quad 1 \;\; \frac{3}{2} \;\; 2 \;\; \frac{5}{2} \;\; 3$

x	$2x-1$		
1	1	1	1
1.5	2	2	4
2	3	2	6
2.5	4	2	8
3	5	1	5
			24

$T = \frac{1}{2}(\text{sum})\underset{\uparrow \, 1/2}{\Delta x} = \frac{1}{2}\cdot 24 \cdot \frac{1}{2} = \underline{\underline{6}}$

Because $f(x) = 2x-1$ is <u>linear</u>, $f''(x)=0$ so $\underline{\underline{|E_T| = 0}}$

the error is <u>zero</u>.

Sec 8.7 #6 $\int_2^4 \frac{1}{(s-1)^2}\,ds$ Trap. rule w/ $n=4$ $\quad 2 \;\; \frac{5}{2} \;\; 3 \;\; \frac{7}{2} \;\; 4$

s	$\frac{1}{(s-1)^2}$		
2	$1 = 1.000$	1	1.000
5/2	$4/9 = 0.444$	2	.888
3	$1/4 = 0.25$	2	.500
7/2	$4/25 = 0.16$	2	.320
4	$1/9 = 0.1111$	1	.111
			2.81999… $= \underline{\underline{2.82}}$

$T = \frac{1}{2}\cdot 2.82 \cdot \frac{1}{2} = .705$

To estimate error: $f(s) = \frac{1}{(s-1)^2}$

$f'(s) = \frac{-2}{(s-1)^3}$

$f''(s) = \frac{6}{(s-1)^4}$

this is a <u>decreasing</u> function, so max f'' on $[2,4]$ is $f''(2) = 6$.

$\Rightarrow |E_T| < \frac{6\cdot 2^3}{12\cdot 4^2} = \frac{1}{4} = 0.25$.

Actual integral is $\int_2^4 \frac{1}{(s-1)^2}\,ds = \left.\frac{-1}{s-1}\right|_2^4 = -\frac{1}{3} + 1 = \frac{2}{3}$

Actual error is $0.705 - 0.6667 = 0.038333$, which is a lot less than 0.25

Percent error: $\frac{0.038333}{2/3} \approx 0.0575$ so about 5.75% error.

Sec 8.8 #2 $\int_1^\infty \frac{dx}{x^{1.001}} = \lim_{b\to\infty}\int_1^b \frac{dx}{x^{1.001}} = \lim_{b\to\infty} \left. -\frac{1}{0.001}\,\frac{1}{x^{0.001}}\right|_1^b$

$= \lim_{b\to\infty} 1000\left[1 - \frac{1}{b^{0.001}}\right] = \underline{\underline{1000}}$

Sec 8.8 # 8

$\int_0^1 \frac{dr}{r^{0.999}} = \lim_{a\to 0^+}\int_a^1 \frac{dr}{r^{0.999}} = \lim_{a\to 0^+} \left.\frac{1}{0.001} r^{0.001}\right|_a^1 = \lim_{a\to 0^+} 1000\left[1 - a^{0.001}\right] = \underline{\underline{1000}}$

Sec 8.8 #16

$$\int_0^2 \frac{s+1}{\sqrt{4-s^2}}\,ds = \lim_{b\to 2^-} \int_0^b \frac{s+1}{\sqrt{4-s^2}}\,ds = \lim_{b\to 2^-} \int_0^{\arcsin b/2} \frac{2\sin\theta+1}{2\cos\theta}\,2\cos\theta\,d\theta$$

$s = 2\sin\theta$
$ds = 2\cos\theta\,d\theta$

$$= \lim_{b\to 2^-} \int_0^{\arcsin b/2} 2\sin\theta + 1\,d\theta = \lim_{b\to 2^-} \theta - 2\cos\theta \Big|_0^{\arcsin b/2}$$

$$= \lim_{b\to 2^-} \arcsin\frac{b}{2} - 2\sqrt{1-\left(\frac{b}{2}\right)^2} + 2 = \frac{\pi}{2} + 2$$

Sec 8.8 #24

$$\int_{-\infty}^{\infty} 2x e^{-x^2}\,dx = \lim_{\substack{b\to\infty \\ a\to-\infty}} \int_a^b 2x e^{-x^2}\,dx = \lim_{\substack{b\to\infty \\ a\to-\infty}} -e^{-x}\Big|_a^b = \lim_{\substack{b\to\infty \\ a\to-\infty}} e^{-a^2} - e^{-b^2} = 0 - 0 = 0$$

Sec 8.8 #48

$$\int_4^{\infty} \frac{dx}{\sqrt{x}-1} > \int_4^{\infty} \frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_4^{\infty} = \infty \text{ so } \underline{\text{diverges}}.$$

OR: $u = \sqrt{x}$
$x = u^2$
$dx = 2u\,du$

$$\int_4^{\infty} \frac{dx}{\sqrt{x}-1} = \int_2^{\infty} \frac{2u\,du}{u-1} = \int_2 2\left(1+\frac{1}{u-1}\right)du = 2u + 2\ln(u-1)\Big|_2^{\infty}$$

$= \infty$ so diverges

Sec 8.8 #65

a) $$\int_1^2 \frac{dx}{x(\ln x)^p} = \int_0^{\ln 2} \frac{du}{u^p} = \begin{cases} \dfrac{u^{1-p}}{1-p}\Big|_0^{\ln 2} & p\neq 1 \to \text{converges if } 1-p>0 \text{ (ie } p<1) \text{, diverges if } 1-p<0 \text{ (ie } p>1) \\ \ln u\Big|_0^{\ln 2} & p=1 \to \text{diverges for } p=1. \end{cases}$$

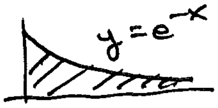
$u = \ln x$
$du = \frac{1}{x}dx$

$\Rightarrow$ Converges if $p<1$, Diverges if $p \geq 1$.

b) $$\int_2^{\infty} \frac{dx}{x(\ln x)^p} = \begin{cases} \dfrac{u^{1-p}}{1-p}\Big|_{\ln 2}^{\infty} & p\neq 1 \text{ converges if } 1-p<0 \text{ (ie. } p>1) \text{, diverges if } 1-p>0 \text{ (ie. } p<1) \\ \ln u & p=1 \to \text{diverges for } p=1 \end{cases}$$

$\Rightarrow$ Converges if $p>1$, Diverges if $p \leq 1$.

Sec 8.8 #68 Centroid: i) area $\int_0^\infty e^{-x}\,dx = -e^{-x}\Big|_0^\infty = 1.$

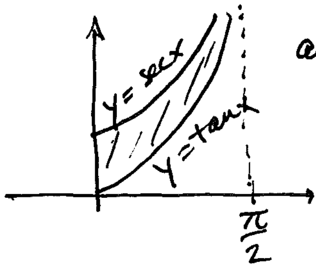


$x \geq 0$

ii) $\bar{x} = \frac{1}{1}\int_0^\infty x e^{-x}\,dx = xe^{-x}\Big|_0^\infty + \int_0^\infty e^{-x}\,dx = 0+1 = \underline{\underline{1}}$

$\bar{y} = \frac{1}{2}\int_0^\infty e^{-2x}\,dx = -\frac{1}{4}e^{-2x}\Big|_0^\infty = \underline{\underline{\frac{1}{4}}}$ Centroid is $(1, \frac{1}{4})$

Sec 8.8 #72



around x-axis

a) Volume $= \pi\int_0^{\pi/2} \sec^2 x - \tan^2 x\,dx = \pi\int_0^{\pi/2} 1\,dx = \frac{\pi^2}{2}.$

b) S.A. of top: $y = \sec x$, $y' = \sec x \tan x$, $y'^2 + 1 = \sec^2 x \tan^2 x + 1.$

$SA = 2\pi\int_0^{\pi/2} y\,ds = 2\pi\int_0^{\pi/2} \sec x\sqrt{\sec^2 x \tan^2 x + 1}\,dx$

$> 2\pi\int_0^{\pi/2} \sec x\sqrt{\sec^2 x \tan^2 x}\,dx = 2\pi\int_0^{\pi/2} \sec^2 x \tan x\,dx$

$(u = \tan x)$ $= 2\pi \frac{\tan^2 x}{2}\Big|_0^{\pi/2} = \underline{\underline{\infty}}.$

S.A. of bottom: $y = \tan x$, $y' = \sec^2 x$ $y'^2 + 1 = \sec^4 x + 1$.

$SA = 2\pi\int_0^{\pi/2} y\,ds = 2\pi\int_0^{\pi/2} \tan x\sqrt{\sec^4 x + 1}\,dx \geq 2\pi\int_0^{\pi/2} \tan x\sqrt{\sec^4 x}\,dx$

$= 2\pi\int_0^{\pi/2} \tan x \sec^2 x\,dx = 2\pi\frac{\tan^2 x}{2}\Big|_0^{\pi/2} = \underline{\underline{\infty}}$

Sec 8.9 #12

$f(x) = \frac{\ln x}{x^2}$ for $x \geq$ this is ≥ 0. $\int_1^\infty \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} + \int_1^\infty \frac{dx}{x^2} = -\frac{\ln x}{x} - \frac{1}{x}\Big|_1^\infty = 1$ ✓

$u = \ln x \to du = \frac{1}{x}dx$ So this is a probability distribution

$dv = \frac{1}{x^2}dx \to v = -\frac{1}{x}$

$P(2 < x < 15) = \int_2^{15} \frac{\ln x}{x^2} = -\frac{\ln x}{x} - \frac{1}{x}\Big|_2^{15} = \frac{\ln 2 + 1}{2} - \frac{\ln 15 + 1}{15} = \underline{\frac{13}{30} + \frac{1}{2}\ln 2 - \frac{1}{15}\ln 15}$

Sec 8.9 #18 $f(x) = \frac{1}{x}$ on $[c, c+1]$ If $c > 0$ then f is positive

$\int_c^{c+1} \frac{1}{x}\,dx = \ln x\Big|_c^{c+1} = \ln(c+1) - \ln c = \ln\frac{c+1}{c} = 1 \Rightarrow \frac{c+1}{c} = e$

or $c+1 = ce \Rightarrow 1 = (e-1)c$

$\Rightarrow c = \frac{1}{e-1}$

Sec 8.9 # 22 $f(x) = c\sqrt{x}\,(1-x)$ on $[0,1]$

need $1 = c\int_0^1 \sqrt{x} - x^{3/2}\,dx = c\left(\frac{2}{3} - \frac{2}{5}\right) = c\cdot\frac{4}{15} \Rightarrow c = \frac{15}{4}$.

$$P\left(\tfrac{1}{4} \le x \le \tfrac{1}{2}\right) = \frac{15}{4}\left(\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right)\Big|_{1/4}^{1/2} = \frac{1}{2}x^{3/2}(5-3x)\Big|_{1/4}^{1/2}$$

$$= \frac{1}{2}\left[\frac{1}{2\sqrt{2}}\left(5-\frac{3}{2}\right) - \frac{1}{8}\left(5-\frac{3}{4}\right)\right] = \frac{1}{2}\left[\frac{7}{4\sqrt{2}} - \frac{17}{32}\right]$$

$$= \frac{7}{16}\sqrt{2} - \frac{17}{64}$$

Sec 8.9 #28 $f(x) = \frac{1}{x}$ $1 \le x \le e$.

Mean: $\mu = \int_1^e x\cdot\frac{1}{x}\,dx = e - 1$

Median $\frac{1}{2} = \int_1^M \frac{1}{x}\,dx = \ln x\Big|_1^M = \ln M \Rightarrow M = e^{1/2} = \sqrt{e}$.

Sec 8.9 #30 $\mu = 4 \Rightarrow f(x) = \frac{1}{4}e^{-x/4}$ with 1000 flowers:

Estimate is $1000\cdot P(0 \le x \le 5) = 1000\cdot\frac{1}{4}\int_0^5 e^{-x/4} = 250\left(-4e^{-x/4}\right)\Big|_0^5$

$$= 1000\left(1 - e^{-1.25}\right) \approx 713.5.$$

Sec 8.9 # 38. Exp dist $f(x) = ce^{-cx}$ on $[0,\infty]$ want Median $= 2$.

So $\int_2^\infty ce^{-cx}\,dx = \frac{1}{2}$ $= -e^{-cx}\Big|_2^\infty = e^{-2c} \Rightarrow -2c = \ln\frac{1}{2} \Rightarrow c = \frac{1}{2}\ln 2$

$$P(x<1) = \int_0^1 \tfrac{1}{2}\ln 2\, e^{-\left(\frac{1}{2}\ln 2\right)x}\,dx = -e^{-\left(\frac{1}{2}\ln 2\right)x}\Big|_0^1 = 1 - e^{-\frac{1}{2}\ln 2} = 1 - \frac{1}{\sqrt{2}}$$

So # failures with 150 copies is $150\left(1 - \frac{1}{\sqrt{2}}\right) \approx 45$

Sec 8.9 # 46 $\mu = 1400$, $\sigma = 100$, $Z = \frac{x-\mu}{\sigma}$

a) $P(1325 < X < 1450) = P(-0.75 < Z < 0.50)$
$= 0.27337 + 0.19146 = 0.46483$ (use table)

b) $P(X > 1480) = P(Z > 0.8) = 1 - 0.71186$
$= 0.28814$

Sec 8.9 # 50 $\mu = 200$, $\sigma = 30$ $P(X > 260 \text{ OR } X < 170) = P(X > 260) + P(X < 170)$

$= P(Z > 2) + P(Z < -1)$

$= 0.0214 + 0.1574 = 0.1788$ (p 524)