Sec. 8.2 p467 #4

$$\int x^2 \sin x\, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$$

D		I
x^2	+	$\sin x$
$2x$	−	$-\cos x$
2	+	$-\sin x$
0		$\cos x$

(or: $u = x^2$ $dv = \sin x\, dx$ etc...)
$du = 2x\, dx$ $v = -\cos x$

Sec. 8.2 p467 #6

$$\int_1^e x^3 \ln x\, dx = \frac{1}{4}x^4 \ln x \Big|_1^e - \int_1^e \frac{1}{4}x^3 dx = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 \Big|_1^e$$

$u = \ln x$ $dv = x^3 dx$

$du = \frac{1}{x}dx$ $v = \frac{1}{4}x^4$

$$= \frac{1}{4}e^4 - \frac{1}{16}e^4 - \left(0 - \frac{1}{16}\right)$$

$$= \frac{3}{16}e^4 + \frac{1}{16}.$$

Sec. 8.2 p467 #12

$$\int \arcsin y\, dy = y \arcsin y - \int \frac{y}{\sqrt{1-y^2}}dy = y\arcsin y + \int \frac{1}{2}\frac{dz}{\sqrt{z}}$$

$u = \arcsin y$ $dv = dy$ | $z = 1 - y^2$ | $= y\arcsin y + \sqrt{z} + C$

$du = \frac{1}{\sqrt{1-y^2}}dy$ $v = y$ | $dz = -2y\, dy$ | $= y\arcsin y + \sqrt{1-y^2} + C$

Sec. 8.2 p467 #20

$$\int t^2 e^{4t} dt = \frac{1}{4}t^2 e^{4t} - \frac{1}{8}te^{4t} + \frac{1}{32}e^{4t} + C$$

D		I
t^2	+	e^{4t}
$2t$	−	$\frac{1}{4}e^{4t}$
2	+	$\frac{1}{16}e^{4t}$
0		$\frac{1}{64}e^{4t}$

(or: $u = t^2$ $dv = e^{4t}$ etc..)
$du = 2t\,dt$ $v = \frac{1}{4}e^{4t}$

Sec 8.2 p467 #32

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = \int 2\cos u\,du = 2\sin u + C$$
$$= 2\sin\sqrt{x} + C$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

Sec 8.2 p467 #46

$$\int \sqrt{x}\,e^{\sqrt{x}}\,dx = \int u\,e^u\,2u\,du = \int 2u^2 e^u = 2u^2e^u - 4ue^u + 4e^u + C$$
$$= 2xe^{\sqrt{x}} - 4\sqrt{x}\,e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$$

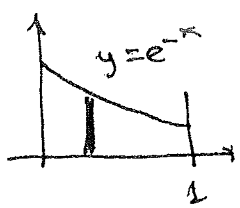
$u = \sqrt{x} \Leftrightarrow x = u^2$

$dx = 2u\,du$

Parts:

D		I
$2u^2$	$+$	e^u
$4u$	$-$	e^u
4	$+$	e^u
0		e^u

Sec 8.2 p467 #56



(a) Volume around y-axis. Shells:

Vol of one shell $= 2\pi\,\underline{r}\,\underline{h}\,\underline{t} = 2\pi\,\underline{x}\,\underline{e^{-x}}\,\underline{dx}$

Total volume $= \int_0^1 2\pi x e^{-x}\,dx = -2\pi x e^{-x}\Big|_0^1 + \int_0^1 2\pi e^{-x}\,dx$

$u = 2\pi x \quad dv = e^{-x}dx$

$du = 2\pi\,dx \quad v = -e^{-x}$

$= -2\pi x e^{-x} - 2\pi e^{-x}\Big|_0^1$

$= -4\pi e^{-1} + 2\pi$

$= 2\pi\left(1 - \frac{2}{e}\right).$

(b) Volume around $x=1$. Shells:

Vol of one shell $= 2\pi\,\underline{r}\,\underline{h}\,\underline{t} = 2\pi\,\underline{(1-x)}\,\underline{e^{-x}}\,\underline{dx}.$

Total volume $= \int_0^1 2\pi(1-x)e^{-x}\,dx = -2\pi(1-x)e^{-x}\Big|_0^1 - \int_0^1 2\pi e^{-x}\,dx$

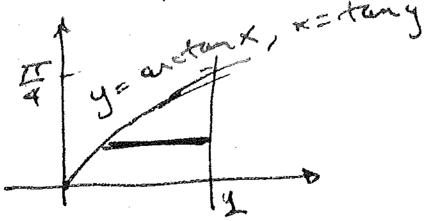
$u = 2\pi(1-x) \quad dv = e^{-x}dx$

$du = -2\pi\,dx \quad v = -e^{-x}$

$= -2\pi(1-x)e^{-x} + 2\pi e^{-x}\Big|_0^1$

$= \frac{2\pi}{e} + 2\pi - 2\pi = \frac{2\pi}{e}.$

Sec 8.2 p467 #60



(a) Area: $\displaystyle\int_0^{\pi/4} 1 - \tan y \, dy = y + \ln(\cos y)\Big|_0^{\pi/4}$

$$= \frac{\pi}{4} + \ln\frac{\sqrt{2}}{2} = \frac{\pi}{4} - \frac{1}{2}\ln 2$$

(b) Volume around y-axis: Washers.

One washer: $\pi R^2 h - \pi r^2 h$

$$= \pi 1^2 dy - \pi \tan^2 y \, dy$$

Total vol: $\displaystyle\int_0^{\pi/4} \pi(1-\tan^2 y)\,dy = \int_0^{\pi/4} \pi(2-\sec^2 y)\,dy$

$$= \pi(2y - \tan y) = \pi\left(\frac{\pi}{2} - 1\right)$$

$$= \frac{\pi^2}{2} - \pi.$$

Sec 8.2 p467 #62

Average of $4e^{-t}(\sin t - \cos t)$ on $[0, 2\pi]$

$$I = \frac{1}{2\pi}\int_0^{2\pi} 4e^{-t}(\sin t - \cos t)\,dt = \frac{2}{\pi}\left[-e^{-t}(\cos t + \sin t)\Big|_0^{2\pi} - \int_0^{2\pi} e^{-t}(\cos t + \sin t)\,dt\right]$$

$u = e^{-t}$ $\quad dv = (\sin t - \cos t)\,dt$

$du = -e^{-t}dt$ $\quad v = -\cos t - \sin t$

$$= \frac{2}{\pi}\left[-e^{-t}(\cos t + \sin t)\Big|_0^{2\pi} - e^{-t}(\sin t - \cos t)\Big|_0^{2\pi} - \int_0^{2\pi} e^{-t}(\sin t - \cos t)\,dt\right].$$

$u = e^{-t}$ $\quad dv = (\cos t + \sin t)\,dt$

$du = -e^{-t}dt$ $\quad v = \sin t - \cos t$

So $\displaystyle I = \frac{2}{\pi}\left[-e^{-t}(\cos t + \sin t) - e^{-t}(\sin t - \cos t)\right]_0^{2\pi} - I$

$$\Rightarrow I = \frac{-1}{\pi}\left[e^{-t}(2\sin t)\right]_0^{2\pi} = \underline{\underline{0}}$$

Sec 8.3 p474 #8

$$\int_0^{\pi} \sin^5 \frac{x}{2}\, dx = \int_0^{\pi} (1-\cos^2\frac{x}{2})^2 \sin\frac{x}{2}\, dx$$

$u = \cos\frac{x}{2}$

$du = -\frac{1}{2}\sin\frac{x}{2}\, dx$

$$= -2\int_1^0 (1-u^2)^2\, du = 2\int_0^1 1-2u^2+u^4\, du = 2\left(u-\frac{2u^3}{3}+\frac{u^5}{5}\right)\Big|_0^1$$

$$= 2\left(1-\frac{2}{3}+\frac{1}{5}\right) = \frac{16}{15}$$

Sec 8.3 p474 #18

$$\int 8\cos^4 2\pi x\, dx = \int 8\,(\cos^2 2\pi x)^2\, dx = \int 8\left(\frac{1+\cos 4\pi x}{2}\right)^2 dx$$

$$= \int 2\,(1+2\cos 4\pi x + \cos^2 4\pi x)\, dx = \int 2 + 4\cos 4\pi x + 1 + \cos 8\pi x\, dx$$

$$= 3x + \frac{1}{\pi}\sin 4\pi x + \frac{1}{8\pi}\sin 8\pi x + C$$

Sec 8.3 p474 # 22

$$\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\, d\theta = \int_0^{\pi/2} \sin^2 2\theta\,(1-\sin^2 2\theta)\cos 2\theta\, d\theta = \int_0^0 \frac{1}{2}u^2(1-u^2)\, du$$

$$= 0$$

$u = \sin 2\theta$

$du = 2\cos 2\theta\, d\theta$

Sec 8.3 p474 # 28

$$\int_0^{\pi/6} \sqrt{1+\sin x}\, dx = \int_0^{\pi/6} \sqrt{1+\sin x}\,\frac{\sqrt{1-\sin x}}{\sqrt{1-\sin x}}\, dx = \int_0^{\pi/6} \frac{\cos x}{\sqrt{1-\sin x}}\, dx$$

$u = 1-\sin x$

$du = -\cos x\, dx$

$$= -\int_1^{1/2} \frac{du}{\sqrt{u}} = 2\sqrt{u}\,\Big|_{1/2}^{1} = 2\left(1-\sqrt{\tfrac{1}{2}}\right)$$

$$= \underline{2-\sqrt{2}}$$

Sec 8.3 p474 #44

$$\int \sec^6 x\,dx = \int \sec^4 x \cdot \sec^2 x\,dx = \int (\tan^2 x + 1)^2 \sec^2 x\,dx$$

$u = \tan x$
$du = \sec^2 x\,dx$

$$= \int (u^2+1)^2\,du = \int u^4 + 2u^2 + 1\,du$$

$$= \frac{1}{5}u^5 + \frac{2}{3}u^3 + u + C$$

$$= \frac{1}{5}\tan^5 x + \frac{2}{3}\tan^3 x + \tan x + C.$$

Sec 8.3 p474 #64

$$\int \frac{\sin^3 x}{\cos^4 x}\,dx = \int \frac{(1-\cos^2 x)\sin x}{\cos^4 x}\,dx = \int \frac{u^2-1}{u^4}\,du$$

$u = \cos x$
$du = -\sin x\,dx$

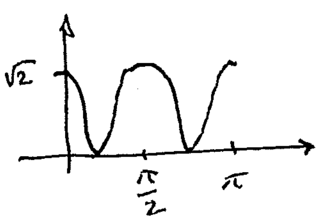
$$= \int \frac{1}{u^2} - \frac{1}{u^4}\,du$$

$$= -\frac{1}{u} + \frac{1}{3u^3} + C$$

$$= \frac{1}{3\cos^3 x} - \frac{1}{\cos x} + C$$

Sec 8.3 p474 #72

$y = \sqrt{1+\cos 4x}$ $\quad 0 \le x \le \pi$



$$\int_0^\pi \sqrt{1+\cos 4x}\,dx = \int_0^\pi \sqrt{2\cos^2 2x}\,dx$$

$$= \sqrt{2}\int_0^\pi |\cos 2x|\,dx = \sqrt{2}\cdot 4\int_0^{\pi/4} \cos 2x\,dx = 2\sqrt{2}\sin 2x\Big|_0^{\pi/4}$$

$$= 2\sqrt{2}$$

Sec 8.4 p 479 #4

$$\int_0^2 \frac{dx}{8+2x^2} = \frac{1}{8}\int_0^2 \frac{dx}{1+\frac{x^2}{4}} = \frac{1}{4}\int_0^{\pi/4} \frac{\sec^2\theta\,d\theta}{1+\tan^2\theta} = \frac{1}{4}\theta\Big|_0^{\pi/4} = \pi/16$$

$\tan\theta = \frac{x}{2}$
$\sec^2\theta\,d\theta = \frac{1}{2}dx$

Sec 8.4 p479 #8

$$\int \sqrt{1-9t^2}\,dt = \frac{1}{3}\int \sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta = \frac{1}{3}\int \cos^2\theta\,d\theta = \frac{1}{6}\int 1+\cos 2\theta\,d\theta$$

$\sin\theta = 3t$

$\cos\theta\,d\theta = 3\,dt$

$$= \frac{1}{6}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C = \frac{1}{6}\theta + \frac{1}{6}\sin\theta\cos\theta + C$$

$$= \frac{1}{6}\arcsin 3t + \frac{1}{6}\cdot 3t\cdot\sqrt{1-9t^2} + C$$

$$= \frac{1}{6}\arcsin 3t + \frac{1}{2}t\sqrt{1-9t^2} + C$$

Sec 8.4 p479 #20

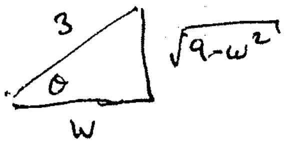
$$\int \frac{\sqrt{9-w^2}}{w^2}\,dw = \int \frac{\sqrt{9-9\cos^2\theta}}{9\cos^2\theta}(-3\sin\theta\,d\theta) = \int -\frac{\sin^2\theta}{\cos^2\theta}\,d\theta = -\int \tan^2\theta\,d\theta$$

$w = 3\cos\theta$

$dw = -3\sin\theta\,d\theta$

$$= \int 1-\sec^2\theta\,d\theta = \theta - \tan\theta + C$$

$$= \arccos\frac{w}{3} - \frac{\sqrt{9-w^2}}{w} + C.$$



Sec 8.4 p479 #32

$$\int \frac{x}{25+4x^2}\,dx = \frac{1}{8}\int \frac{du}{u} = \frac{1}{8}\ln u + C = \frac{1}{8}\ln(25+4x^2) + C$$

$u = 25+4x^2$

$du = 8x\,dx$

Sec 8.4 p479 #36

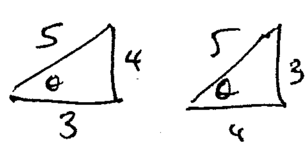
$$\int_{\ln 3/4}^{\ln 4/3} \frac{e^t\,dt}{(1+e^{2t})^{3/2}} = \int_{3/4}^{4/3} \frac{du}{(1+u^2)^{3/2}} = \int_{\arctan 3/4}^{\arctan 4/3} \frac{\sec^2\theta\,d\theta}{\sec^3\theta} = \int_{\arctan 3/4}^{\arctan 4/3} \cos\theta\,d\theta = \sin\theta\Big|_{\text{arctan}}^{\text{arctan}}$$

$u = e^t$

$du = e^t\,dt$

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$



$$= \frac{4}{5} - \frac{3}{5} = \underline{\underline{\frac{1}{5}}}$$

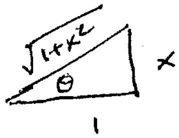
Sec 8.4 p479 # 52

$(x^2+1)^2 \frac{dy}{dx} = \sqrt{x^2+1} \qquad y(0)=1$

$x = \tan\theta \quad dx = \sec^2\theta\, d\theta$

$$\frac{dy}{dx} = \frac{1}{(x^2+1)^{3/2}} \Rightarrow y = \int \frac{dx}{(x^2+1)^{3/2}} = \int \frac{\sec^2\theta\, d\theta}{\sec^3\theta} = \int \cos\theta\, d\theta = \sin\theta + C$$

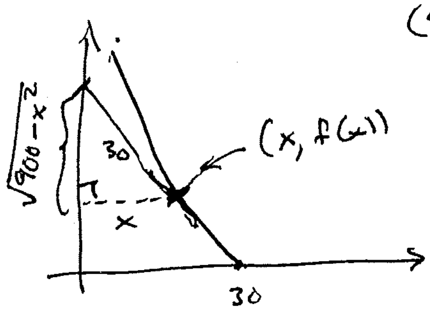
$$= \frac{x}{\sqrt{1+x^2}} + C$$



$y(0)=1: \quad 0 + C = 1 \Rightarrow C = 1$

$$\boxed{y = \frac{x}{\sqrt{1+x^2}} + 1}$$

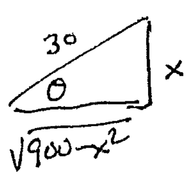
Sec 8.4 p479 # 58



(a) Slope of tangent line at $(x, f(x))$

$$= f'(x) = -\frac{\sqrt{900-x^2}}{x} \quad \text{by Pythagorean thm.}$$

(b) $f(x) = -\int \frac{\sqrt{900-x^2}}{x}\, dx \qquad x = 30\sin\theta, \quad dx = 30\cos\theta\, d\theta$



$$= -\int \frac{30\cos\theta}{30\sin\theta} \cdot 30\cos\theta\, d\theta$$

$$= -\int \frac{30(1-\sin^2\theta)}{\sin\theta} d\theta = -30 \int \csc\theta - \sin\theta\, d\theta$$

$$= 30\ln(\csc\theta + \cot\theta) - 30\cos\theta + C$$

$$= 30\ln\left(\frac{30}{x} + \frac{\sqrt{900-x^2}}{x}\right) - 30\frac{\sqrt{900-x^2}}{30} + C$$

$$= 30\ln\left(30 + \sqrt{900-x^2}\right) - 30\ln(x) - \sqrt{900-x^2} + C$$

$f(30)=0: \quad 30\ln 30 - 30\ln 30 - 0 + C = 0 \Rightarrow C = 0$

$$y = 30\ln(30 + \sqrt{900-x^2}) - 30\ln x - \sqrt{900-x^2}$$