

Name _____

Recitation Time _____

Math 104 SAIL: Midterm III
Tuesday, December 6th, 2016
12:00–1:20
DRL 3N1H

This exam has 12 multiple choice questions worth 6 points each. Circle the correct answer(s) (worth one point each) and give supporting work (worth up to five points). To be clear: correct answers with no supporting work will receive only one out of six possible points. **Circle your answer for each problem and use the space provided to show all work;** you may write on the back side of the pages if necessary, but be sure to clearly indicate what work you would like to have graded. A sheet of scrap paper is also provided at the end of the exam.

You have 80 minutes to complete the exam. You are not allowed the use of books, notebooks, calculators, or any other electronic devices. You are allowed to use the front and back of an 8.5"×11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. If you finish after 1:10, you must remain in your seat until time has elapsed. When time is up, please stay seated until someone comes by to collect your exam.

Problem	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	6	
8	6	
9	6	
10	6	
11	6	
12	6	
Total	72	

1. Find the y -coordinate of the centroid of the region bounded by the curves $y = x$ and $y = x^2$.

(a) $-\frac{2}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{12}$ (d) $-\frac{1}{2}$ (e) $\frac{1}{2}$ (f) $\frac{1}{4}$

2. Find the interval of convergence of the power series below. Justify all parts of your answer.

$$\sum_{n=1}^{\infty} \frac{2^{n+1}(x+3)^n}{3^n(n+1)^2}$$

- (a) $\left(-\frac{9}{2}, -\frac{3}{2}\right)$ (b) $\left[-\frac{9}{2}, -\frac{3}{2}\right]$ (c) $(-5, -1)$ (d) $[-5, 1]$ (e) $(-\infty, \infty)$ (f) none of the above

3. Suppose that $\sum_{n=0}^{\infty} a_n$ converges to 2. Complete the following statement and justify your response: The series $\sum_{n=0}^{\infty} e^{a_n}$
- (a) converges to 1. (b) converges to 2. (c) converges to 2^e .
- (d) converges to e^2 . (e) diverges. (f) none of the above

4. Find the constant C so that the function

$$f(x) = C\sqrt{x-1}$$

is a probability density function on the interval $[1, 2]$. Then compute the mean μ .

(a) $C = 4, \mu = \frac{9}{5}$ (b) $C = \frac{3}{2}, \mu = \frac{3}{5}$ (c) $C = 2, \mu = 1$

(d) $C = 3, \mu = \frac{8}{5}$ (e) $C = \frac{3}{2}, \mu = \frac{7}{5}$ (f) $C = \frac{3}{2}, \mu = \frac{8}{5}$

5. The region in the plane bounded by $y = e^{-x/2}$ and the x -axis for $0 \leq x \leq \ln 2$ is rotated about the x -axis. The volume of the resulting solid of revolution is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{3}{2}$ (d) 2π (e) $\frac{5}{3}$ (f) $\frac{\pi}{2}$

6. Compute the definite integral

$$\int_0^{\pi} \sin^3 x \cos^4 x \, dx.$$

- (a) 0 (b) $\frac{4}{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{4}{35}$ (e) $\sqrt{6}$ (f) none of the above

7. Let $y(x)$ be the solution to the initial value problem $x\frac{dy}{dx} - 2y = x^3$ and $y(1) = 0$. What is $y(3)$?

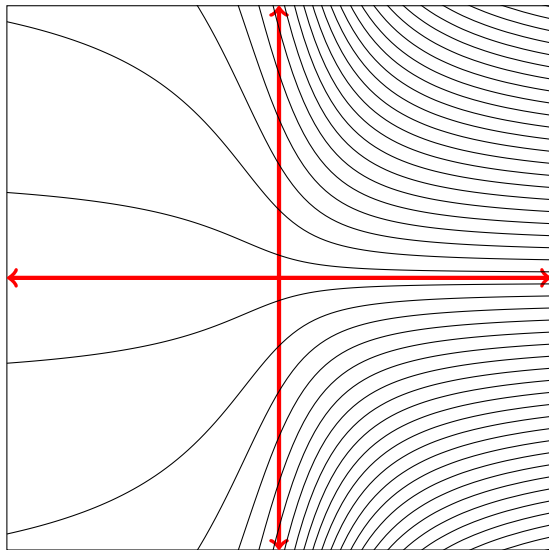
- (a) 1 (b) 3 (c) 6 (d) 9 (e) 12 (f) none of the above

8. Evaluate the indefinite integral

$$\int \frac{2}{y^3 + y} dy.$$

- (a) $2 \ln(y^3 + y) + C$ (b) $\ln\left(\frac{y^2}{y^2 + 1}\right) + C$ (c) $\frac{1}{y^2 + \ln y} + C$
(d) $\frac{1}{(y^3 + y)^2} + C$ (e) $\ln(2y^2 + 1) + C$ (f) $2 \ln |y| = y + C$

9. The family of curves $y = Ce^{-\arctan x}$, where C is a constant which can have any real value, is plotted below. Find the formula for the orthogonal trajectories and sketch at least two of these new curves on the plot.



- (a) $y' = -\frac{y}{1+x^2}$ (b) $yy' = 1+x^2$ (c) $y = -Ce^{\arctan x}$
 (d) $x^2+y^2 = C$ (e) $\frac{y^2}{2} = x + \frac{x^3}{3} + C$ (f) none of the above

10. Compute the value of the integral below.

$$\int_1^{\sqrt{2}} \frac{\sqrt{x^2 - 1}}{x} dx$$

- (a) $1 - \frac{\pi}{4}$ (b) $2 - \frac{\pi}{2}$ (c) $4 - \pi$ (d) $6 - \frac{3\pi}{2}$ (e) $8 - 2\pi$ (f) 0

11. Determine whether each series converges absolutely, conditionally, or diverges. Justify both your answers.

$$\text{I: } \sum_{n=1}^{\infty} \frac{1}{n + n^2 e^{-n}} \quad \text{II: } \sum_{n=1}^{\infty} \frac{1}{n + n^2 e^n}$$

- (a) I and II both diverge (b) I and II both converge conditionally (c) I and II both converge absolutely
(d) I diverges, II converges absolutely (e) II diverges, I converges absolutely (f) none of the above

12. Give an integral formula for the area of the surface generated by revolving the curve $y = \ln x$ between $x = 1$ and $x = 2$ about the y -axis. Explain your answer. You do not need to evaluate the integral.

$$\begin{array}{lll} \text{(a)} \int_1^2 2\pi\sqrt{x^2+1} \, dx & \text{(b)} \int_1^2 2\pi(\ln x)\frac{\sqrt{x^2+1}}{x} \, dx & \text{(c)} \int_1^2 \frac{2\pi}{x}\sqrt{1+(\ln x)^2} \, dx \\ \text{(d)} \int_1^2 \frac{1}{2\pi\sqrt{x^2+1}} \, dx & \text{(e)} \int_1^2 2\pi(\ln x)\frac{x}{\sqrt{x^2+1}} \, dx & \text{(f)} \int_1^2 \frac{2\pi x}{\sqrt{1+(\ln x)^2}} \, dx \end{array}$$

