Name _____

Recitation Time _____

Math 104 SAIL: Midterm II
Tuesday, November 8th, 2016
12:00–1:20
DRL 3N1H

This exam has 12 multiple choice questions worth 6 points each. Circle the correct answer(s) (worth one point each) and give supporting work (worth up to five points). To be clear: correct answers with no supporting work will receive only one out of six possible points. **Circle your answer for each problem and use the space provided to show all work**; you may write on the back side of the pages if necessary, but be sure to clearly indicate what work you would like to have graded. A sheet of scrap paper is also provided at the end of the exam.

You have 80 minutes to complete the exam. You are not allowed the use of books, notebooks, calculators, or any other electronic devices. You are allowed to use the front and back of an 8.5"×11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. If you finish after 1:10, you must remain in your seat until time has elapsed. When time is up, please stay seated until someone comes by to collect your exam.

Problem	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	6	
8	6	
9	6	
10	6	
11	6	
12	6	
Total	72	

1. Evaluate the integral $\displaystyle\int_1^3 \left(x - \sqrt{4x^2 - 8x + 13}\right) dx$ using the fact that $\displaystyle\int_0^4 \sqrt{x^2 + 9}\, dx = \frac{20 + 9\ln 3}{2}$.

(a) $-\dfrac{4 + 9\ln 3}{6}$ (b) $-\dfrac{4 + 9\ln 3}{4}$ (c) $-\dfrac{4 + 9\ln 3}{2}$ (d) $-4 - 9\ln 3$ (e) $-8 - 18\ln 3$ (f) $-12 - 27\ln 3$

2. How many equally spaced intervals N are sufficient for the trapezoidal rule to estimate the value of the following integral with an error less than 10^{-6}? Choose the smallest value of N among the options listed which would be sufficient.

$$\int_{-1}^{1} e^{x^2-1} dx.$$

(a) 2 (b) 20 (c) 200 (d) 2000 (e) 20000 (f) none of the above

3. Determine whether the following improper integrals are convergent or divergent. Justify your answers.

$$\text{I: } \int_0^\infty \frac{dt}{2e^t + 1} \qquad \text{II: } \int_0^1 \frac{dx}{x^2}$$

(a) both convergent (b) both divergent (c) only I converges (d) only II converges

(e) impossible to determine (f) none of the above

4. Suppose X is a random variable which represents the amount of time (in minutes) that workers at a 911 call center have to wait between consecutive calls. Assume that for every value of $t > 0$,

$$P(t \leq X < \infty) = e^{-\frac{t}{2}}.$$

Compute the expected value of X.

(a) $\mu = 1$ (b) $\mu = 2$ (c) $\mu = 3$ (d) $\mu = 4$ (a) $\mu = 5$ (f) none of the above

In the context of this specific example, briefly explain what it means to say that this distribution is memoryless.

5. State the formal definition of the limit of a sequence. Then, for the sequence below, find a lower bound for n which is sufficient when $\epsilon = \frac{1}{402}$. If your own answer differs from the multiple choice option you select, explain the apparent contradiction.

$$\lim_{n\to\infty} \frac{n^2}{2n^2+1}$$

(a) $n > 1$ (b) $n > 2$ (c) $n > 4$ (d) $n > 8$ (e) $n > 16$ (f) none of the above

6. Determine the value of the following limit:

$$\lim_{n\to\infty}\left[\frac{n+1}{2n+\sin n}-\left(1-\frac{\ln 2}{n}\right)^n\right].$$

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2 (f) does not exist

7. What does the integral test say about the following series?

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

(a) test doesn't apply but series converges anyway (b) test applies and series converges

(c) test doesn't apply but series diverges anyway (d) test applies and series diverges

(e) test doesn't apply; series doesn't converge or diverge (f) test applies; series doesn't converge or diverge

8. Determine whether these series converge or diverge. Justify both your answers.

$$\text{I: } \sum_{n=1}^{\infty} \frac{1}{e^n - e^{-n}} \qquad \text{II: } \sum_{n=1}^{\infty} \frac{(-1)^n}{e^n - e^{-n}}$$

(a) both convergent (b) both divergent (c) only I converges (d) only II converges

(e) impossible to determine (f) none of the above

9. Verify that the function $f(x)$ given below is a probability density function on the interval $[1, 100]$. Compute the median of the distribution. What is the relationship between the median and the mean (i.e., expected value)? Justify all parts of your answer.

$$f(x) := \frac{1}{\ln(100)} \frac{1}{x}$$

(a) median $= 10$, mean is less than median

(b) median $= 10$, mean is equal to median

(c) median $= 10$, mean is greater than median

(d) median $= \dfrac{99}{\ln(100)}$, mean is less than median

(e) median $= \dfrac{99}{\ln(100)}$, mean is equal to median

(f) median $= \dfrac{99}{\ln(100)}$, mean is greater than median

10. Two of the following three sequences are convergent. In each case, determine whether or not the limit exists and, when possible, evaluate the limit. Briefly describe your reasoning for each answer.

$$a_1 = \frac{9}{10} \text{ and } a_{n+1} = \frac{1}{2}a_n(a_n + 1) \text{ for } n > 1.$$
$$b_1 = \frac{10}{10} \text{ and } b_{n+1} = \frac{1}{2}b_n(b_n + 1) \text{ for } n > 1.$$
$$c_1 = \frac{11}{10} \text{ and } c_{n+1} = \frac{1}{2}c_n(c_n + 1) \text{ for } n > 1.$$

(a) $a_n \to 0,\ b_n \to 1,\ c_n \to \infty$ (b) $a_n \to 1,\ b_n \to 1,\ c_n \to \infty$ (c) $a_n \to \infty,\ b_n \to 1,\ c_n \to 0$

(d) $a_n \to \infty,\ b_n \to 1,\ c_n \to 1$ (e) $a_n \to 0,\ b_n \to \infty,\ c_n \to 1$ (f) none of the above

11. Determine the convergence or divergence of the following improper integral, and select the answer below which correctly justifies your answer.

$$\int_0^1 \ln|x^2 - 1| dx.$$

(a) Limit comparison to $\ln|x+1|$ implies convergence (b) Limit comparison to $\ln|x+1|$ implies divergence

(a) Limit comparison to $\ln|x-1|$ implies convergence (b) Limit comparison to $\ln|x-1|$ implies divergence

(a) Limit comparison to $\ln|x^2|$ implies convergence (b) Limit comparison to $\ln|x^2|$ implies divergence

12. Determine the convergence or divergence of the following series, and select the answer below which correctly justifies your answer.

$$\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)}{n}$$

(a) diverges by ratio test (b) converges by ratio test (c) diverges by root test

(d) converges by root test (e) converges by alternating series test (f) none of the above