

Name _____

Recitation Time _____

Math 104 SAIL: Midterm I
Thursday, September 29th, 2016
12:00–1:20
DRL 3N1H

This exam has 12 multiple choice questions worth 6 points each. Circle the correct answer(s) (worth one point each) and give supporting work (worth up to five points). To be clear: correct answers with no supporting work will receive only one out of six possible points. **Circle your answer for each problem and use the space provided to show all work;** you may write on the back side of the pages if necessary, but be sure to clearly indicate what work you would like to have graded. A sheet of scrap paper is also provided at the end of the exam.

You have 80 minutes to complete the exam. You are not allowed the use of books, notebooks, calculators, or any other electronic devices. You are allowed to use the front and back of an 8.5"×11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. If you finish after 1:10, you must remain in your seat until time has elapsed. When time is up, please stay seated until someone comes by to collect your exam.

Problem	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	6	
8	6	
9	6	
10	6	
11	6	
12	6	
Total	72	

1. Which of the following integrals equals the area of the surface of revolution obtained by rotating the curve $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{2}$ around the y -axis? **You do not need to compute the integral.**

(a) $\int_0^{\frac{\pi}{2}} 2\pi \sin x \sqrt{1+x^2} \, dx$ (b) $\int_0^{\frac{\pi}{2}} 2\pi \cos x \sqrt{1+x^2} \, dx$ (c) $\int_0^{\frac{\pi}{2}} \pi \cos^2 x \, dx$

(d) $\int_0^{\frac{\pi}{2}} 2\pi x \sqrt{1+\sin^2 x} \, dx$ (e) $\int_0^{\frac{\pi}{2}} 2\pi x \cos x \, dx$ (f) $\int_0^{\frac{\pi}{2}} \sqrt{1+\sin^2 x} \, dx$

2. Which of the following is the centroid of the region given by $x^2 + y^2 \leq 1$, $x \geq 0$, and $y \geq 0$? Justify your response.

- (a) $\left(\frac{4}{3\pi}, \frac{4}{3\pi}\right)$ (b) $\left(\frac{1}{2}, \frac{4}{3\pi}\right)$ (c) $\left(\frac{4}{3\pi}, \frac{1}{2}\right)$ (d) $\left(\frac{4}{3\pi}, \frac{5}{3\pi}\right)$ (e) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (f) $\left(\frac{5}{3\pi}, \frac{4}{3\pi}\right)$

Without doing the calculation, is $\bar{x}$ greater than or less than $\frac{1}{2}$? Give a brief geometric explanation.

3. Compute the value of the following integral. Be sure to justify your answer.

$$\int_0^1 x \arctan x \, dx$$

- (a) $\frac{\pi}{16} - \frac{1}{8}$ (b) $\frac{\pi}{8} - \frac{1}{4}$ (c) $\frac{\pi}{4} - \frac{1}{2}$ (d) $\frac{\pi}{2} - 1$ (e) $\pi - 2$ (f) none of the above

4. Suppose that some sand at ground level (with density ρ kilograms per cubic meter) is collected and packed into a solid sphere, resting on the ground, with radius one half of a meter. What is the minimum amount of energy this activity would require? Use g for gravitational acceleration (your final answer should be in Joules).

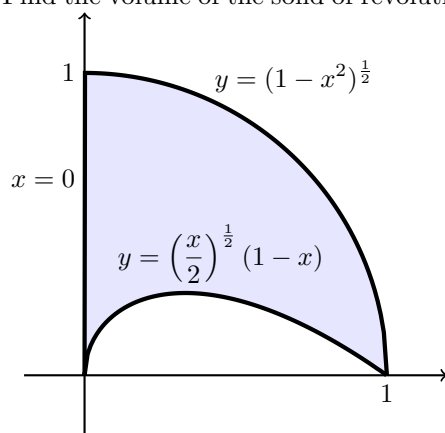
(a) $\frac{\pi\rho g}{25}$ (b) $\frac{\pi\rho g}{12}$ (c) $\frac{\pi\rho g}{7}$ (d) $\frac{\pi\rho g}{3}$ (e) $\pi\rho g$ (f) none of the above

5. Compute the integral

$$\int \sec x \tan^3 x \, dx.$$

- (a) $\sec^3 x + C$ (b) $\tan^3 x + C$ (c) $\frac{\sec^3 x}{3} + C$
(d) $\frac{\tan^3 x}{3} + C$ (e) $\frac{\sec^3 x}{3} - \sec x + C$ (f) $\frac{\tan^3 x}{3} - \tan x + C$

6. Find the volume of the solid of revolution obtained by rotating the planar region indicated below around the axis $y = 0$.



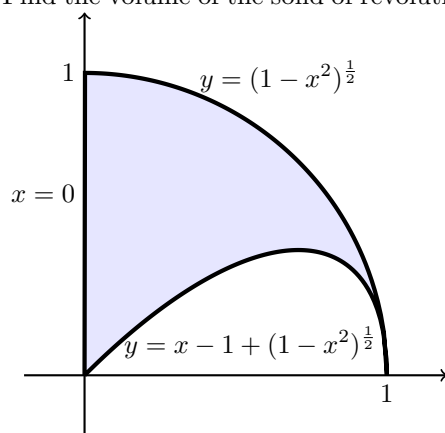
- (a) $\frac{\pi}{8}$ (b) $\frac{2\pi}{8}$ (c) $\frac{3\pi}{8}$ (d) $\frac{4\pi}{8}$ (e) $\frac{5\pi}{8}$ (f) $\frac{6\pi}{8}$

7. Compute the integral below. Be sure to show your work.

$$\int \frac{\cos \ln |x|}{x} dx$$

- (a) $\cos \ln |x| + C$ (b) $\sin \ln |x| + C$ (c) $\tan \ln |x| + C$ (d) $\sec \ln |x| + C$ (e) $\csc \ln |x| + C$ (f) $\cot \ln |x| + C$

8. Find the volume of the solid of revolution obtained by rotating the planar region indicated below around the axis $x = 2$.



- (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{3}$ (d) $\frac{4\pi}{3}$ (e) $\frac{5\pi}{3}$ (f) $\frac{6\pi}{3}$

Without doing the calculation, if the axis were changed to $x = -1$, would the volume increase or decrease? Give a brief explanation of your reasoning.

9. Compute the value of the integral below. Be sure to show your work.

$$\int_0^4 \frac{dx}{(x^2 + 9)^{\frac{3}{2}}}$$

- (a) $\frac{1}{8}$ (b) $\frac{4}{15}$ (c) $\frac{3}{8}$ (d) $\frac{16}{35}$ (e) $\frac{25}{24}$ (f) none of the above

10. Expand the following expression using partial fractions:

$$\frac{x^2 - 4x + 5}{x^2 - 3x + 2}.$$

$$\begin{array}{lll} \text{(a)} \ 1 + \frac{2}{5(x-1)} - \frac{17}{5(x-2)} & \text{(b)} \ 1 - \frac{2}{x-1} + \frac{1}{x-2} & \text{(c)} \ 1 + \frac{3}{x-1} - \frac{5}{x-2} \\ \text{(d)} \ -1 + \frac{2}{5(x-1)} - \frac{17}{5(x-2)} & \text{(e)} \ -\frac{2}{x-1} + \frac{1}{x-2} & \text{(f)} \ 2 + \frac{3}{x-1} - \frac{5}{x-2} \end{array}$$

11. Find the arclength of the following curve between $x = 0$ and $x = 1$:

$$y = \frac{x^3 + 3x}{4} - \frac{1}{3} \arctan x.$$

- (a) $\frac{\pi}{12} + 1$ (b) $\frac{\pi}{12} - 1$ (c) $1 + \frac{1}{3} \ln 2$ (d) $1 - \frac{1}{3} \ln 2$ (e) $\frac{\pi}{12} + \frac{1}{3} \ln 2$ (f) none of the above

12. Compute the value of the integral below.

$$\int_1^e \frac{(x^2 + 3x + 1) \ln |x|}{x(x+1)^2} dx$$

(a) $\frac{1}{e+1}$

(b) $\ln(e+1)$

(c) $\frac{3}{2}$

(d) $\frac{3}{2}$

(e) $\frac{3}{2} - \frac{1}{e+1}$

(f) $\frac{3}{2} - \frac{1}{e+1} - \ln(e+1) + \ln 2$

