

$$11. \int_0^1 (1+\sqrt{x})^2 dx = \int_1^2 u^2 \cdot 2(u-1) du$$

$$\begin{aligned} u &= 1+\sqrt{x} \\ x &= (u-1)^2 \\ dx &= 2(u-1) du \end{aligned} \quad \left| \begin{aligned} &= \int_1^2 2u^2 \cdot 2(u-1) du \\ &= \frac{1}{5} u^{10} - \frac{2}{9} u^9 \Big|_1^2 \end{aligned} \right.$$

$$\begin{aligned} &= \frac{1024}{5} - \frac{1024}{9} - \frac{1}{5} + \frac{2}{9} \\ &= \frac{4096}{45} + \frac{1}{45} = \frac{4097}{45} \end{aligned}$$

$$\begin{aligned} 16. \int \frac{x}{x^2+3x+2} dx &= \int \frac{x}{(x+2)(x+1)} dx \\ &= \int \frac{2}{x+2} - \frac{1}{x+1} dx \\ &= 2 \ln|x+2| - \ln|x+1| + C \end{aligned}$$

$$17. \int x^2 \cosh x dx = x^2 \sinh x - 2x \cosh x + 2 \sinh x + C$$

Part: $\begin{array}{l} D \quad I \\ x^2 \quad \cosh x \\ 2x \quad \sinh x \\ 2 \quad \cosh x \\ 0 \quad \sinh x \end{array}$

$$12. \int_0^{\pi/4} \tan^3 x \sec^4 x dx = \int_0^{\pi/4} \tan^2 x (\tan^2 x + 1) \sec^2 x dx$$

$$\begin{aligned} u &= \tan x \\ du &= \sec^2 x dx \\ &= \int_0^1 u^3 (u^2 + 1) du \\ &= \int_0^1 (u^5 + u^3) du = \frac{1}{6} u^6 + \frac{1}{4} u^4 \Big|_0^1 \\ &= \frac{1}{6} + \frac{1}{4} = \frac{10}{24} = \frac{5}{12} \end{aligned}$$

$$\begin{aligned} 18. \int \frac{x^3+x+1}{x^4+2x^2+4x} dx &= \frac{1}{4} \int \frac{du}{u} \\ u &= x^4+2x^2+4x \\ du &= 4x^3+4x+4 \end{aligned} \quad \left| \begin{aligned} &= \frac{1}{4} \ln|u| + C \\ &= \frac{1}{4} \ln|x^4+2x^2+4x| + C \end{aligned} \right.$$

$$\begin{aligned} 13. \int \frac{x}{x^2-2x+2} dx &= \int \frac{x-1}{x^2-2x+2} dx + \int \frac{1}{x^2-2x+2} dx \\ u &= x^2-2x+2 \\ &= \frac{1}{2} \int \frac{du}{u} + \int \frac{1}{(x-1)^2+1} dx \\ &= \frac{1}{2} \ln|u| + \arctan(x-1) + C \\ &= \frac{1}{2} \ln(x^2-2x+2) + \arctan(x-1) + C \end{aligned}$$

$$\begin{aligned} 19. \int \frac{\cos x}{1+\sin^2 x} dx &= \int \frac{du}{1+u^2} \\ u &= \sin x \\ du &= \cos x dx \end{aligned} \quad \left| \begin{aligned} &= \arctan u + C \\ &= \arctan(\sin x) + C \end{aligned} \right.$$

$$\begin{aligned} 20. \int \cos \sqrt{x} dx &= \int (\cos u) 2u du \\ u &= \sqrt{x}, \quad x=u^2 \\ dx &= 2u du \end{aligned} \quad \left| \begin{aligned} &= 2u \sin u + 2 \cos u + C \\ &= 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C \end{aligned} \right.$$

$$\begin{aligned} 14. \int x \sin^{-1} x dx &= \int x \arcsin x dx \\ \text{Part: } u &= \arcsin x, \quad du = \frac{dx}{\sqrt{1-x^2}} \\ v &= \frac{1}{2} x^2 \\ &= \frac{1}{2} x^2 \arcsin x - \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx \\ &= \frac{1}{2} x^2 \arcsin x - \frac{1}{2} \int \sin^2 \theta d\theta \\ &= \frac{1}{2} x^2 \arcsin x - \frac{1}{4} \int (1 - \cos 2\theta) d\theta \\ &= \frac{1}{2} x^2 \arcsin x - \frac{1}{4} \theta + \frac{1}{8} \sin 2\theta + C \\ &= \frac{1}{2} x^2 \arcsin x - \frac{1}{4} \arcsin x + \frac{1}{4} \sqrt{1-x^2} + C \end{aligned}$$

$$\begin{aligned} 15. \int \frac{\sqrt{9-x^2}}{x} dx &= \int \frac{3 \cos \theta \cdot 3 \sin \theta d\theta}{3 \sin \theta} = 3 \int \frac{1-\sin^2 \theta}{\sin \theta} d\theta = 3 \int (\csc \theta - \sin \theta) d\theta = 3(-\ln|\csc \theta + \cot \theta| + \cos \theta) + C \\ x &= 3 \sin \theta \\ dx &= 3 \cos \theta d\theta \\ \sin \theta &= \frac{x}{3} \end{aligned} \quad \left| \begin{aligned} &= 3(-\ln|\frac{3}{x} + \frac{\sqrt{9-x^2}}{x}| + \frac{\sqrt{9-x^2}}{3}) + C \\ &= \sqrt{9-x^2} - 3 \ln|\frac{3+\sqrt{9-x^2}}{x}| + C \end{aligned} \right.$$

