1. $\int \frac{2x+5}{x-3}\,dx = \int 2 + \frac{11}{x-3}\,dx$

$= 2x + 11\ln|x-3| + C$

2. $\int e^{x+e^x}\,dx = \int e^{e^x}\cdot e^x\,dx$

$u = e^x$, $du = e^x dx$

$= \int e^u\,du$

$= e^u + C = e^{e^x} + C$

3. $\int \sin^2 x \cos^3 x\,dx = \int \sin^2 x(1-\sin^2 x)\cos x\,dx$

$u = \sin x$, $du = \cos x\,dx$

$= \int u^2(1-u^2)\,du$

$= \int u^2 - u^4\,du$

$= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$= \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$

4. $\int \frac{\sin x - \cos x}{\sin x + \cos x}\,dx = \int \frac{-du}{u}$

$u = \sin x + \cos x$

$du = \cos x - \sin x\,dx$

$= -\ln|u| + C$

$= -\ln|\sin x + \cos x| + C$

5. $\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int_1^{3/4} \frac{du}{\sqrt{u}} = \frac{1}{2}\cdot 2\,u^{1/2}\Big|_{3/4}^{1}$

$u = 1-x^2$

$du = -2x\,dx$

$= 1 - \frac{\sqrt{3}}{2}$.

6. $\int_1^2 x^3 \ln x\,dx = \frac{1}{4}x^4\ln x - \int \frac{1}{4}x^4\cdot\frac{1}{x}\,dx$

Parts: $u = \ln x$, $dv = x^3dx$; $du = \frac{1}{x}dx$, $v = \frac{1}{4}x^4$

$= \frac{1}{4}x^4\ln x - \frac{1}{4}\int x^3\,dx$

$= \frac{1}{4}x^4\ln x - \frac{1}{16}x^4\Big|_1^2$

$= \frac{16}{4}\ln 2 - \frac{16}{16} - \left(0 - \frac{1}{16}\right)$

$= 4\ln 2 - \frac{15}{16}$

7. $\int \frac{\sqrt{x-2}}{x+2}\,dx = \int \frac{u}{u^2+4}\,2u\,du$

$u = \sqrt{x-2}$

$du = \frac{1}{2\sqrt{x-2}}\,dx$

$x = u^2+2$

$dx = 2u\,du$

$= \int \frac{2u^2}{u^2+4}\,du = \int 2 - \frac{8}{u^2+4}\,du$

$= 2u - \frac{8}{4}\int \frac{1}{\left(\frac{u}{2}\right)^2+1}\,du$

$\tan\theta = \frac{u}{2}$

$\sec^2\theta\,d\theta = \frac{1}{2}du$

$= 2u - 2\int \frac{2\sec^2\theta\,d\theta}{\sec^2\theta}$

$= 2u - 4\theta + C$

$= 2\sqrt{x-2} - 4\arctan\frac{\sqrt{x-2}}{2} + C$

8. $\int \frac{x}{(x+2)^2}\,dx = \int \frac{u-2}{u^2}\,du$

$u = x+2$

$du = dx$

$(x = u-2)$

$= \int \frac{1}{u} - \frac{2}{u^2}\,du$

$= \ln|u| + \frac{2}{u} + C$

$= \ln|x+2| + \frac{2}{x+2} + C$

9. $\int \ln(1+x^2)\,dx = x\ln(1+x^2) - \int \frac{2x^2}{1+x^2}\,dx$

Parts: $u = \ln(1+x^2)$, $dv = dx$; $du = \frac{2x}{1+x^2}dx$, $v = x$

$= x\ln(1+x^2) - 2\int 1 - \frac{1}{1+x^2}\,dx$

$= x\ln(1+x^2) - 2x + 2\arctan x + C$

10. $\int \frac{\sqrt{1+\ln x}}{x\ln x}\,dx = \int \frac{\sqrt{1+u}}{u}\,du$

$u = \ln x$

$du = \frac{1}{x}dx$

$v = \sqrt{1+u}$

$u = v^2 - 1$

$du = 2v\,dv$

$= \int \frac{v\cdot 2v\,dv}{v^2-1} = \int \frac{2v^2}{v^2-1}\,dv$

$= \int 2 + \frac{2}{(v-1)(v+1)}\,dv$

$= \int 2 + \frac{1}{v-1} - \frac{1}{v+1}\,dv = 2v + \ln|v-1| - \ln|v+1| + C$

$= 2\sqrt{1+u} + \ln\left|\frac{\sqrt{1+u}-1}{\sqrt{1+u}+1}\right| + C$

$= 2\sqrt{1+\ln x} + \ln\left|\frac{\sqrt{1+\ln x}-1}{\sqrt{1+\ln x}+1}\right| + C$