Math 646, Problem set 4, due November 6, 2003
Dr. Epstein

Read sections 2.8, 2.9, 2.11 and 3.1 of the notes. Hormander pages 44-52.

1. Suppose that $u \in P(\Omega)$, i.e. u is plurisubharmonic in Ω. Given $\phi \in \mathscr{C}_c^\infty(\mathbb{C}^n)$, a nonnegative function of $\{|z_1|, \dots, |z_n|\}$, with support in the ball of radius 1, which satisfies
$$\int \phi \, \mathrm{dV} = 1,$$
show that:
   (a) The functions
   $$u_\epsilon(z) = \int u(z + \epsilon w)\phi(w) \, \mathrm{dV},$$
   are smooth and plurisubharmonic in
   $$\Omega_\epsilon = \{z \in \epsilon; d(z, \Omega^c) > \epsilon\}.$$
   (b) $u_\epsilon(z) \geq u_\delta(z)$ if $\epsilon > \delta$. Hint: Show that for a subharmonic function, u the averages:
   $$\overline{u}_r(x) = \frac{1}{|bB(x,r)|} \int\limits_{bB(x,r)} u(y) dS(y),$$
   are a nondecreasing function of r.
   (c) $\lim_{\epsilon \to 0} u_\epsilon = u$.
2. Suppose that Ω is a bounded pseudoconvex domain in $\mathbb{C}^n$. Show that there is a sequence of strictly pseudoconvex subdomains $\Omega_1 \subset\subset \Omega_2 \subset\subset \cdots \subset \Omega$ such that $\partial\Omega_i$ is smooth and $\cup \Omega_i = \Omega$.
3. Exercise 2.8.30 on page 30 of the notes.
4. Let $\Omega \subset \mathbb{C}^n$ be a smoothly bounded domain with defining function ρ. If $f \in \mathscr{C}^\infty(\partial\Omega)$ show that $\overline{Z} f = 0$ for all sections, $\overline{Z}$ of $T^{0,1}\partial\Omega$ is equivalent to
$$\bar{\partial} f \wedge \bar{\partial}\rho \restriction_{\partial\Omega} = 0.$$
In the second statement one must extend f to a neighborhood of $\partial\Omega$, so you also need to show that $\bar{\partial} f \wedge \bar{\partial}\rho \restriction_{\partial\Omega}$ is independent of the extension.
5. Exercise 2.12.3 on page 44 of the notes.
6. Exercise 2.12.14 on page 44 of the notes.