

Math 646, Problem set 3 due October 21, 2003

Dr. Epstein

Read sections 2.7, 2.8 and 2.9 and appendix B of Chapter 2 of the notes. Hormander pages 31-44.

1. Prove: If u_1 and u_2 are subharmonic functions defined in a domain in \mathbb{C} then

$$u(z) = \max\{u_1(z), u_2(z)\}$$

is also subharmonic. Conclude that if v_1 and v_2 are plurisubharmonic functions defined on a domain in \mathbb{C}^n then

$$v(z) = \max\{v_1(z), v_2(z)\}$$

is also plurisubharmonic.

2. A \mathcal{C}^2 -function f , defined in an open set $U \subset \mathbb{C}^n$, is called pluriharmonic if its restriction to any complex line is harmonic.

(a) Show that a function f is pluriharmonic if and only if

$$\frac{\partial^2 f}{\partial z_j \partial \bar{z}_k} = 0 \text{ for all } j, k.$$

(b) Show that a real valued function is pluriharmonic if and only if it is locally the real part of a holomorphic function.

3. Exercise 2.8.6 in the notes.
4. Let $\Omega \subset \mathbb{C}^n$ be a domain. An open subset A of Ω is called an analytic polyhedron, if there is a finite set of functions $\{f_1, \dots, f_N\} \subset H(\Omega)$ such that

$$A = \{z \in \Omega : |f_j(z)| < 1 \text{ for } j = 1, \dots, N\}.$$

Suppose that Ω is a domain of holomorphy and $K \subset\subset \Omega$ then there is an analytic polyhedron A such that

$$K \subset\subset A \subset\subset \Omega.$$

5. Suppose that V is the common zero set of a collection of holomorphic functions $\{f_j(z), j = 1, \dots, N\}$ defined in \mathbb{C}^N . Show that V has an open neighborhood which is pseudoconvex.
6. A submanifold M of \mathbb{C}^n is totally real if $JTM \cap TM = \{0\}$. Here J denote the standard complex structure on \mathbb{C}^n . Show that, if M is a compact totally real submanifold, then, for sufficiently small $\epsilon > 0$, the open sets

$$\Omega_\epsilon = \{z : d(z, M) < \epsilon\}$$

are strictly pseudoconvex.