Math 646, Problem set 2 due October 7, 2003

Dr. Epstein

Read sections 2.1, 2.2, 2.4, 2.5, and 2.6 of Chapter 2 of the notes, skip section 2.3. Hormander pages 22-36.

1. Exercise 2.1.5
2. Exercises 2.1.23.
3. Exercises 2.A.2 (page 9 of Appendix A)
4. Exercise 2.4.4.
5. Use the argument principle for holomorphic functions of 1-variable to give a proof that a holomorphic function of several variables cannot have isolated zeros.
6. Let $f \in H(D_1 \times D_1)$, show that the hypothesis $f(z_1, z_2) \neq 0$ for $\{(z_1, z_2) : |z_1| < \frac{1}{2} \text{ or } |z_2| < \frac{1}{2}\}$ implies that f does not vanish on the unit ball.
7. Let $\Omega = \{(z_1, z_2) : |z_1| < |z_2| < 1\}$. Prove that every function holomorphic in a neighborhood of $\overline{\Omega}$ has a holomorphic extension to the polydisk $D_1 \times D_1$.
8. Let
$$\Omega = \{(z_1, z_2) : \frac{1}{4} < |z_2| < 1, |z_1| < 1\} \cup \{(z_1, z_2) : |z_2| < \frac{1}{2}, |z_1| < \frac{1}{2}\}.$$
Find the image of Ω under the map $(z_1, z_2) \to (\log|z_1|, \log|z_2|)$ and show that Ω is not a complete Reinhardt domain. Find the smallest complete Reinhardt domain which contains Ω.