Math 584, Problem set 2 due February 14, 2017
Dr. Epstein

Reading: Read sections 3.1-3.4. You are free to use Mathematica, or MATLAB etc. to do these problems, but please turn in copies of the program's output.

1. For functions that are integrable on $[0, 1]$ we define the function, taking values in $[0, \infty)$,
$$\|f\|_1 = \int_0^1 |f(x)|dx.$$
Show that $\| \cdot \|_1$ defines a norm on the set of functions continuous on the interval $[0, 1]$. Construct a sequence of continuous functions $< f_n >$ such that there is a *discontinuous* function, f, which is nonetheless integrable, (see Theorem B.6.1) such that
$$\lim_{n\to\infty} \|f_n - f\|_1 = 0.$$
2. Show that if
$$f(x) = g(x) + \int_0^x e^{(x-y)} g(y)dy,$$
then f solves the equation:
$$f(x) - \int_0^x f(y)dy = g(x).$$
3. Do problems 2.2.4, 2.2.5 from the text.
4. Do problem 3.1.1 from the text.
5. Do problem 3.2.1 from the text. For this problem, explain why the top and bottom are identical, up to a reflection.
6. Do problems 3.4.2, 3.4.8 from the text.
7. Define a function $f(x, y)$ by the formula
$$f(x, y) = \begin{cases} 1 - (x^2 + y^2) \text{ if} & x^2 + y^2 < 1, \\ 0 \text{ if} & x^2 + y^2 \geq 1. \end{cases}$$
Compute its Radon transform $\mathrm{R}(f)$.
8. Exercises 3.4.9 from the text.