Math 508

Problem set 4, due October 2, 2018

Dr. Epstein

Your solutions to these problems should be written in English: Use complete sentences and paragraphs.

For this week, the material comes from sections 3.1 and section 7.2 of **The Way of Analysis**. Standard problems, whose solutions do not need to be handed in.

1. Let A and B be sets of real numbers. Show that
$$\sup(A \cup B) \geq \sup A \text{ and } \sup(A \cap B) \leq \sup A.$$
2. Prove that every subsequence of a subsequence of a sequence is also a subsequence of the original sequence.

The following problems should be carefully written up and handed in:

1. Let $< x_n >$ be a sequence of real numbers. Prove that
$$\limsup_{n\to\infty}(-x_n) = -\liminf_{n\to\infty} x_n.$$
2. Compute the sup, inf, lim sup, lim inf of the following sequences
   (a) $x_n = \frac{1}{n} + (-1)^n$.
   (b) $x_n = 1 + \frac{(-1)^n}{n}$.
   (c) $x_n = (-1)^n + \frac{1}{n} + 2\sin(\frac{n\pi}{2})$.
3. Suppose that $< x_n >$ and $< y_n >$ are sequences with finite lim sups. Show that
$$\limsup_{n\to\infty}(x_n + y_n) \leq \limsup_{n\to\infty} x_n + \limsup_{n\to\infty} y_n.$$
   Give an example to show that this inequality is sometimes strict.
4. Find a sequence of numbers whose set of limits points equals the positive integers.
5. Can there exist a sequence whose set of limit points *equals* the set:
$$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}?$$
   Why or why not?
6. Suppose that $\sum_{j=1}^{\infty} a_j$ is convergent, but not absolutely convergent. Let
$$a_j^+ = \max\{a_j, 0\} \text{ and } a_j^- = \min\{a_j, 0\}. \tag{1}$$
   Show that
$$\sum_{j=1}^{\infty} a_j^+ = \infty \text{ and } \sum_{j=1}^{\infty} a_j^- = -\infty. \tag{2}$$