O and o Notation

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People seemed confused by the meaning of O and o notation.

We say that, for any real number k,

$$f(x) = O(|x - x_0|^k) \tag{1}$$

if there are positive constants δ and M so that

$$|f(x)| \leq M|x - x_0|^k \text{ provided } |x - x_0| < \delta. \tag{2}$$

This implies that

$$\limsup_{x \to x_0} \frac{|f(x)|}{|x - x_0|^k} \leq M, \tag{3}$$

but (2) is a little stronger as we do not exclude x_0.

We say that, for any real number k,

$$f(x) = o(|x - x_0|^k) \tag{4}$$

if for any $0 < \epsilon$ there is a $0 < \delta$ so that

$$|f(x)| \leq \epsilon|x - x_0|^k \text{ provided } |x - x_0| < \delta. \tag{5}$$

This implies that

$$\lim_{x \to x_0} \frac{|f(x)|}{|x - x_0|^k} = 0. \tag{6}$$

but (5) is also a little stronger as we do not exclude x_0.