Consider Rock Paper Scissors:

```
R  P  S
R  0, 0 -1, 1 1, -1
P  1, -1 0, 0 -1, 1
S -1, 1 1, -1 0, 0
```

What is the expected payoff of (1/3, 1/3, 1/3) against (1/2, 0, 1/2)?

Note that the expected payout is a weighted average of payouts.
Consider Rock Paper Scissors:

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<tr>
<th></th>
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What is the expected payoff of \((\frac{1}{3}, \frac{1}{3}, \frac{1}{3})\) against \((\frac{1}{2}, 0, \frac{1}{2})\)?
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What is the expected payoff of $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ against $(\frac{1}{2}, 0, \frac{1}{2})$?

0
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Note that the expected payout is a weighted average of payouts
Rock Paper Scissors

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How can we raise the expected payout against (1/2, 0, 1/2)?

By removing pure strategies that lower the average

\[
u((0, 1, 0), (1/2, 0, 1/2)) = 1/2
\]

\[
u((0, 0, 1), (1/2, 0, 1/2)) = -1/2
\]

Best strategy against (1/2, 0, 1/2) is to play rock

A best response to a strategy will consist of pure strategies that have the same (high) expected payout.
Rock Paper Scissors

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- How can we raise the expected payout against \( \left( \frac{1}{2}, 0, \frac{1}{2} \right) \)?

- By removing pure strategies that lower the average expected payout:

\[
\begin{align*}
  u((1/2, 0, 1/2), (1/2, 0, 1/2)) &= 1/2 \\
  u((0, 1, 0), (1/2, 0, 1/2)) &= 0 \\
  u((0, 0, 1), (1/2, 0, 1/2)) &= -1/2
\end{align*}
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By removing pure strategies that lower the average
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R & P & S \\
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S & -1,1 & 1,-1 & 0,0 \\
\end{array}
\]

- How can we raise the expected payout against \((\frac{1}{2}, 0, \frac{1}{2})\)?
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Rock Paper Scissors

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\[
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\text{R} & \text{P} & \text{S} \\
\hline
\text{R} & (0, 0) & -1, 1 & 1, -1 \\
\hline
\text{P} & 1, -1 & 0, 0 & -1, 1 \\
\hline
\text{S} & -1, 1 & 1, -1 & 0, 0 \\
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  - \( u((1, 0, 0), \left( \frac{1}{2}, 0, \frac{1}{2} \right)) = \frac{1}{2} \)
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Best strategy against \( \left( \frac{1}{2}, 0, \frac{1}{2} \right) \) is to play rock
Rock Paper Scissors

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Nash Equilibrium

- Mixed strategies \((p_1, \ldots, p_n)\) are a **Nash equilibrium** if \(p_i\) is a best response to \(p_{-i}\).
Nash Equilibrium

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  - Each player asks “if the other players stuck with their strategies, am I better off mixing the ratio of strategies?”

Note that pure Nash equilibria are still Nash equilibria.
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  - If \(p_i\) is a best response to \(p_{-i}\), the payouts of the pure strategies in \(p_i\) are equal

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What should the (unique) Nash equilibrium be?

Test this: what is the payoff of a pure strategy against $(1/3,1/3,1/3)$?

Note that the expected payoff for each player is 0 (the game is fair).
Rock Paper Scissors

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What should the (unique) Nash equilibrium be?

When both players use \((\frac{1}{3}, \frac{1}{3}, \frac{1}{3})\)
Rock Paper Scissors

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What should the (unique) Nash equilibrium be?

- When both players use \(\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\)
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What should the (unique) Nash equilibrium be?

- When both players use \( \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \)
- Test this: what is the payoff of a pure strategy against \( \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \)?
  - 0

Note that the expected payoff for each player is 0 (the game is fair)
Nash Equilibrium

- We saw that there are not always pure Nash equilibrium
Nash Equilibrium

- We saw that there are not always pure Nash equilibrium
- Can we guarantee a mixed Nash equilibrium?
Nash Equilibrium

- We saw that there are not always pure Nash equilibrium
- Can we guarantee a mixed Nash equilibrium?

**Theorem (Nash)**

Suppose that:

- a game has finitely many players
- each player has finitely many pure strategies
- we allow for mixed strategies

Then the game admits a Nash equilibrium
Tennis

- You’re playing tennis, and returning the ball
Tennis

- You’re playing tennis, and returning the ball
- Options:

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>You L</td>
<td>50,50</td>
<td></td>
</tr>
<tr>
<td>You R</td>
<td></td>
<td>90,10</td>
</tr>
<tr>
<td></td>
<td>20,80</td>
<td></td>
</tr>
<tr>
<td>Opponent L</td>
<td></td>
<td>80,20</td>
</tr>
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<td></td>
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</tbody>
</table>

What are the Nash equilibrium?

- No pure Nash equilibrium

Assume that strategies are $(p, 1-p)$ and $(q, 1-q)$

Idea: your opponent's pure strategies must return the same expected payoff (for them) against $(p, 1-p)$

Strategies in Nash equilibrium are $(.7, .3)$ and $(.6, .4)$.
Tennis

▶ You’re playing tennis, and returning the ball
▶ Options:
  ▶ You can hit the ball to either the opponent’s left or right

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- You’re playing tennis, and returning the ball
- Options:
  - You can hit the ball to either the opponent’s left or right
  - Opponent can anticipate where you will hit the ball (to their left or right)
You’re playing tennis, and returning the ball

Options:
- You can hit the ball to either the opponent’s left or right
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<tr>
<th></th>
<th>Opponent</th>
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<tbody>
<tr>
<td></td>
<td>L</td>
</tr>
<tr>
<td>You</td>
<td>50,50</td>
</tr>
<tr>
<td></td>
<td>90,10</td>
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  - Strategies in Nash equilibrium are \((.7, .3)(p = .7)\) and \((.6, .4)(q = .6)\)
What are the Nash equilibria?
What are the Nash equilibria?

Pure Nash equilibria occur when you both go to the same movie

For mixed Nash equilibria, write out the strategies as 
\((p, 1 - p)\) and \((q, 1 - q)\)
Going to the Movies

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So $2p = p + 3(1 - p)$

$p = \frac{3}{4}$

Similarly, $q = \frac{1}{4}$

Note that both you and your date's expected payout is $\frac{3}{2}$ (between the original payouts of the Nash equilibrium)
Going to the Movies

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- Simplification of theory due to John Maynard Smith
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- Will those with the mutation thrive or die?
Evolutionarily Stable Strategies

- An example: ants may or may not help defend the nest
Evolutionarily Stable Strategies

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- This creates a game such as:

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- Percent of population with mutation will grow until Nash equilibrium is reached