Review

From last time:

- Strategy *A strictly dominates* strategy *B* if the payoff from *A* is higher than the payoff of *B*, regardless of others’ strategies.
Review

From last time:

▶ Strategy A **strictly dominates** strategy B if the payoff from A is higher than the payoff of B, regardless of others’ strategies

▶ **Moral:** you should never pick a dominated strategy
From last time:

- **Strategy** *A strictly dominates* strategy *B* if the payoff from *A* is higher than the payoff of *B*, regardless of others’ strategies.
- **Moral**: you should never pick a dominated strategy.
- **Moral**: If you don’t have a dominated strategy, try to predict your opponents’ choice.
Notation

- Let 1, 2, ..., n denote players
Notation

- Let 1, 2, . . . , n denote players
- Let $s_i$ denote a particular strategy of player $i$
Notation

- Let 1, 2, ..., n denote players
- Let $s_i$ denote a particular strategy of player $i$
- Let $S_i$ denote the set of all strategies of player $i$
Notation

- Let 1, 2, \ldots, n denote players
- Let $s_i$ denote a particular strategy of player $i$
- Let $S_i$ denote the set of all strategies of player $i$
- Let $s_{-i}$ denote a choice of strategy for all players except player $i$
Notation

- Let 1, 2, \ldots, n denote players
- Let $s_i$ denote a particular strategy of player $i$
- Let $S_i$ denote the set of all strategies of player $i$
- Let $s_{-_i}$ denote a choice of strategy for all players except player $i$
- Let $u_i(s_i, s_{-_i})$ denote the utility/payoff for player $i$ if players choose strategies $s_i/s_{-_i}$
Notation

- Let 1, 2, . . . , n denote players
- Let $s_i$ denote a particular strategy of player $i$
- Let $S_i$ denote the set of all strategies of player $i$
- Let $s_{-i}$ denote a choice of strategy for all players except player $i$
- Let $u_i(s_i, s_{-i})$ denote the utility/payoff for player $i$ if players choose strategies $s_i/s_{-i}$
- So $s_i$ strictly dominates $s_i^*$ if $u_i(s_i, s_{-i}) > u_i(s_i^*, s_{-i})$ for all choices of $s_{-i}$
Sacking Rome

- Hannibal wants to cross into Italy with two batallions.
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
- There are two options:
  - Easy path along the coast
  - Hard path through the Alps
- If he takes the hard path, he loses one batallion (just from crossing)
- If he meets the defending army, he loses one batallion

Is this a game?

Need payoffs - let's use # of batallions brought into country and # of batallions die
Hannibal wants to cross into Italy with two batallions.

There are two options:

- Easy path along the coast
- Hard path through the Alps

If he takes the hard path, he loses one batallion (just from crossing).

If he meets the defending army, he loses one batallion.

Is this a game?

Need payoffs - let's use # of batallions brought into country and # of batallions die.
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
- There are two options:
  - Easy path along the coast
  - Hard path through the Alps
Hannibal wants to cross into Italy with two batallions

There are two options:

- Easy path along the coast
- Hard path through the Alps

If he takes the hard path, he loses one batallion (just from crossing)
Hannibal wants to cross into Italy with two batallions

There are two options:
  - Easy path along the coast
  - Hard path through the Alps

If he takes the hard path, he loses one batallion (just from crossing)

If he meets the defending army, he loses one batallion
Sacking Rome

▶ Hannibal wants to cross into Italy with two batallions
▶ There are two options:
  ▶ Easy path along the coast
  ▶ Hard path through the Alps
▶ If he takes the hard path, he loses one batallion (just from crossing)
▶ If he meets the defending army, he loses one batallion
▶ Is this a game?
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
- There are two options:
  - Easy path along the coast
  - Hard path through the Alps
- If he takes the hard path, he loses one batallion (just from crossing)
- If he meets the defending army, he loses one batallion
- Is this a game?
  - Need payoffs - let’s use # of batallions brought into country and # of batallions die
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
- There are two options:
  - Easy path along the coast
  - Hard path through the Alps
  - If he takes the hard path, he loses one batallion (just from crossing)
  - If he meets the defending army, he loses one batallion
- Is this a game?
  - Need payoffs - let's use # of batallions that die/are brought into Italy
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
- There are two options:
  - Easy path along the coast
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
- There are two options:
  - Easy path along the coast
  - Hard path through the Alps
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
- There are two options:
  - Easy path along the coast
  - Hard path through the Alps
- If he takes the hard path, he loses one batallion (just from crossing)
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
- There are two options:
  - Easy path along the coast
  - Hard path through the Alps
- If he takes the hard path, he loses one batallion (just from crossing)
- If he meets the defending army, he loses one batallion
Hannibal wants to cross into Italy with two batallions

There are two options:
- Easy path along the coast
- Hard path through the Alps

If he takes the hard path, he loses one batallion (just from crossing)
If he meets the defending army, he loses one batallion
Is this a game?
Sacking Rome

- Hannibal wants to cross into Italy with two batallions
- There are two options:
  - Easy path along the coast
  - Hard path through the Alps
- If he takes the hard path, he loses one batallion (just from crossing)
- If he meets the defending army, he loses one batallion
- Is this a game?
  - Need payoffs - let’s use # of batallions that die/are brought into Italy
Sacking Rome

The outcome matrix is:

<table>
<thead>
<tr>
<th></th>
<th>Hannibal</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fabius</td>
<td>E</td>
<td>1,1</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>1,1</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>0,2</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>2,0</td>
</tr>
</tbody>
</table>

▶ You are Fabius Maximus. What should you do?
## Sacking Rome

The outcome matrix is:

<table>
<thead>
<tr>
<th></th>
<th>Hannibal</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fabius</td>
<td>E</td>
<td>1,1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>1,1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>E</td>
<td></td>
<td>0,2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H</td>
<td></td>
<td></td>
<td>2,0</td>
</tr>
</tbody>
</table>

- You are Fabius Maximus. What should you do?
  - Are there any dominant strategies?

**Moral**: you should probably never pick a weakly dominated strategy.
You are Fabius Maximus. What should you do?

Are there any dominant strategies?

No strictly dominant strategies.
Sacking Rome

The outcome matrix is:

<table>
<thead>
<tr>
<th></th>
<th>Hannibal</th>
<th>Fabius</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>1, 1</td>
<td>1, 1</td>
</tr>
<tr>
<td>H</td>
<td>0, 2</td>
<td>2, 0</td>
</tr>
</tbody>
</table>

▶ You are Fabius Maximus. What should you do?

▶ Are there any dominant strategies?
  ▶ No strictly dominant strategies
  ▶ Hannibal has a weakly dominant strategy
    He'll probably choose to take the easy route
## Sacking Rome

The outcome matrix is:

|       | Hannibal
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Fabius</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>1,1</td>
</tr>
</tbody>
</table>

- You are Fabius Maximus. What should you do?
  - Are there any dominant strategies?
    - No strictly dominant strategies
      - Hannibal has a weakly dominant strategy
        - He'll probably choose to take the easy route
  - \( s_i \) weakly dominates \( s_i^* \) if \( u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i}) \) for all choices of \( s_{-i} \)
You are Fabius Maximus. What should you do?
  Are there any dominant strategies?
    No strictly dominant strategies
    Hannibal has a weakly dominant strategy
      He'll probably choose to take the easy route

\( s_i \) weakly dominates \( s_i^* \) if \( u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i}) \) for all choices of \( s_{-i} \)

What does this mean?
Sacking Rome

The outcome matrix is:

```
     E    H
  E  1,1  1,1
  H  0,2  2,0
```

▶ You are Fabius Maximus. What should you do?
  ▶ Are there any dominant strategies?
    ▶ No strictly dominant strategies
    ▶ Hannibal has a weakly dominant strategy
      He'll probably choose to take the easy route
  ▶ $s_i$ weakly dominates $s_i^*$ if $u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i})$ for all choices of $s_{-i}$
  ▶ What does this mean?
    ▶ Strategy $s_i$ **weakly dominates** strategy $s_i^*$ if the payoff from $s_i$ is never worse than the payoff of $s_i^*$, regardless of others’ strategies
Sacking Rome

The outcome matrix is:

<table>
<thead>
<tr>
<th></th>
<th>Hannibal</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fabius</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>1, 1</td>
<td>1,1</td>
</tr>
<tr>
<td>H</td>
<td>0,2</td>
<td>2,0</td>
</tr>
</tbody>
</table>

▶ You are Fabius Maximus. What should you do?
▶ Are there any dominant strategies?
  ▶ No strictly dominant strategies
  ▶ Hannibal has a weakly dominant strategy
    He’ll probably choose to take the easy route
▶ \( s_i \) weakly dominates \( s_i^* \) if \( u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i}) \) for all choices of \( s_{-i} \)
▶ What does this mean?
  ▶ Strategy \( s_i \) weakly dominates strategy \( s_i^* \) if the payoff from \( s_i \) is never worse than the payoff of \( s_i^* \), regardless of others’ strategies
▶ Moral: you should probably never pick a weakly dominated strategy
The Numbers Game

Review:

Everyone in class wrote down their name and a number between 1 and 100
The Numbers Game

Review:

- Everyone in class wrote down their name and a number between 1 and 100
- I took \( \frac{2}{3} \) the average of everyones’ numbers
The Numbers Game

Review:

- Everyone in class wrote down their name and a number between 1 and 100
- I took $\frac{2}{3}$ the average of everyone's numbers
- Whoever is closest wins $5 - (0.01) \cdot \text{(how far they are off)}$
The Numbers Game

Review:

- Everyone in class wrote down their name and a number between 1 and 100
- I took \( \frac{2}{3} \) the average of everyone’s numbers
- Whoever is closest wins $5 - (.01) \cdot (\text{how far they are off})
- Example:
The Numbers Game

Review:

► Everyone in class wrote down their name and a number between 1 and 100
► I took $\frac{2}{3}$ the average of everyone's numbers
► Whoever is closest wins $5 - (0.01) \cdot (\text{how far they are off})$
► Example:
  ► Suppose that there are three people in class
The Numbers Game

Review:

▷ Everyone in class wrote down their name and a number between 1 and 100
▷ I took $\frac{2}{3}$ the average of everyones’ numbers
▷ Whoever is closest wins $5 - (.01) \cdot$ (how far they are off)
▷ Example:
  ▸ Suppose that there are three people in class
  ▸ They choose 5, 30, 55
Review:

- Everyone in class wrote down their name and a number between 1 and 100
- I took \( \frac{2}{3} \) the average of everyone’s numbers
- Whoever is closest wins \( \$5 - (.01) \cdot (\text{how far they are off}) \)

Example:

- Suppose that there are three people in class
- They choose 5, 30, 55
- \( \frac{2}{3} \cdot \frac{5 + 30 + 55}{3} = 20 \)
Review:

- Everyone in class wrote down their name and a number between 1 and 100
- I took \( \frac{2}{3} \) the average of everyone's numbers
- Whoever is closest wins $5 - (0.01) \cdot (\text{how far they are off})$
- Example:
  - Suppose that there are three people in class
  - They choose 5, 30, 55
  - \( \frac{2}{3} \cdot \frac{5 + 30 + 55}{3} = 20 \)
  - 30 wins $4.90
The Numbers Game

- Why might someone choose \( \approx 33 \)?

  - If everyone else chooses randomly, the average will be \( \approx 50 \).
  - Two thirds of the average will be \( \approx 33 \).

  - Any problems? If most people think this way, the average will be \( \approx 33 \), and so two thirds of the average will be \( \approx 22 \).
The Numbers Game

- Why might someone choose $\approx 33$?
  - If everyone else chooses randomly, the average will be $\approx 50$
The Numbers Game

Why might someone choose $\approx 33$?

- If everyone else chooses randomly, the average will be $\approx 50$
- Two thirds of the average will be $\approx 33$

Any problems?
The Numbers Game

- Why might someone choose ≈ 33?
  - If everyone else chooses randomly, the average will be ≈ 50
  - Two thirds of the average will be ≈ 33

- Any problems?
  - If most people think this way, the average will be ≈ 33, and so two thirds of the average will be ≈ 22
Iterative Deletion of Dominated Strategies

- Assume:
  - Every player is completely rational
Iterative Deletion of Dominated Strategies

Assume:
- Every player is completely rational
- Every player assumes every other player is completely rational
Iterative Deletion of Dominated Strategies

- Assume:
  - Every player is completely rational
  - Every player assumes every other player is completely rational
- Are any strategies weakly dominated?
Iterative Deletion of Dominated Strategies

▶ Assume:
  ▶ Every player is completely rational
  ▶ Every player assumes every other player is completely rational
▶ Are any strategies weakly dominated?
  ▶ 68 through 100
Iterative Deletion of Dominated Strategies

- Assume:
  - Every player is completely rational
  - Every player assumes every other player is completely rational
- Are any strategies weakly dominated?
  - 68 through 100
  - Cross these strategies out for everybody
Iterative Deletion of Dominated Strategies

- Assume:
  - Every player is completely rational
  - Every player assumes every other player is completely rational
- Are any strategies weakly dominated?
  - 68 through 100
    - Cross these strategies out for everybody
- After crossing these strategies, are any strategies weakly dominated?
Iterative Deletion of Dominated Strategies

- Assume:
  - Every player is completely rational
  - Every player assumes every other player is completely rational

- Are any strategies weakly dominated?
  - 68 through 100
  - Cross these strategies out for everybody

- After crossing these strategies, are any strategies weakly dominated?
  - 46 through 67
Iterative Deletion of Dominated Strategies

- Assume:
  - Every player is completely rational
  - Every player assumes every other player is completely rational
- Are any strategies weakly dominated?
  - 68 through 100
  - Cross these strategies out for everybody
- After crossing these strategies, are any strategies weakly dominated?
  - 46 through 67
- If we continue this process, everyone is left with choosing 1
The class’ numbers were:
1, 1, 1, 2.14, 15, 15, 17.9, 18, 30, 32, 34, 34, 37, 45, 45, 48, 48, 53, 80, 89

Two thirds of the average is 21.53

Congratulations Andre Serrano ($4,80)
The Numbers Game

- The class’ numbers were:
  1, 1, 1, 2.14, 15, 15, 17.9, 18, 30, 32, 34, 34, 37, 45, 45, 48, 48, 53, 80, 89
- Two thirds of the average is 21.53
The Numbers Game

- The class’ numbers were: 1, 1, 1, 2.14, 15, 15, 17.9, 18, 30, 32, 34, 34, 37, 45, 45, 48, 48, 53, 80, 89
- Two thirds of the average is 21.53
- Congratulations Andre Serrano ($4.80)
The Numbers Game

Let’s play this game again
Let’s play this game again

Write down a number between 1 and 100
The Numbers Game

- Let’s play this game again
- Write down a number between 1 and 100
- Who wrote a lower number than last time?
The Numbers Game

- Let’s play this game again
- Write down a number between 1 and 100
- Who wrote a lower number than last time?
- What changed?

Strategies for the game became common knowledge:
1. “Everyone knows the strategy”
2. “Everyone knows that everyone knows the strategy”
3. “Everyone knows that everyone knows that everyone knows the strategy”
   ...
The Numbers Game

- Let’s play this game again
- Write down a number between 1 and 100
- Who wrote a lower number than last time?
- What changed?
  - Strategies for the game became **common knowledge**: 
    - 1. “Everyone knows the strategy”
    - 2. “Everyone knows that everyone knows the strategy”
    - 3. “Everyone knows that everyone knows that everyone knows the strategy”
    - ...
The Numbers Game

- Let’s play this game again
- Write down a number between 1 and 100
- Who wrote a lower number than last time?
- What changed?
  - Strategies for the game became **common knowledge**:
    1. “Everyone knows the strategy”
The Numbers Game

- Let’s play this game again
- Write down a number between 1 and 100
- Who wrote a lower number than last time?
- What changed?
  - Strategies for the game became common knowledge:
    1. “Everyone knows the strategy”
    2. “Everyone knows that everyone knows the strategy”
The Numbers Game

Let’s play this game again

Write down a number between 1 and 100

Who wrote a lower number than last time?

What changed?

Strategies for the game became common knowledge:

1. “Everyone knows the strategy”
2. “Everyone knows that everyone knows the strategy”
3. “Everyone knows that everyone knows that everyone knows the strategy”
Let’s play this game again

Write down a number between 1 and 100

Who wrote a lower number than last time?

What changed?

Strategies for the game became common knowledge:

1. “Everyone knows the strategy”
2. “Everyone knows that everyone knows the strategy”
3. “Everyone knows that everyone knows that everyone knows the strategy”
4. :}
Political Spectrum

Assume:

- There is a spectrum of 10 points on a certain political issue

- There are two candidates

- 10% of the voters hold each position

- Voters will vote for the candidate who holds the closest views

- If the candidates hold the same view, they’ll split the vote

Is this a game?

- Have players (the candidates)

- Have strategies (1 to 10)

- Need payoffs: choose the % of the vote that they earn
Political Spectrum

Assume:

▷ There is a spectrum of 10 points on a certain political issue
▷ There are two candidates
Political Spectrum

Assume:
- There is a spectrum of 10 points on a certain political issue
- There are two candidates
- 10% of the voters hold each position
Political Spectrum

Assume:

- There is a spectrum of 10 points on a certain political issue
- There are two candidates
- 10% of the voters hold each position
- Voters will vote for the candidate who holds the closest views
Political Spectrum

Assume:

- There is a spectrum of 10 points on a certain political issue
- There are two candidates
- 10% of the voters hold each position
- Voters will vote for the candidate who holds the closest views
- If the candidates hold the same view, they’ll split the vote
Political Spectrum

Assume:

- There is a spectrum of 10 points on a certain political issue
- There are two candidates
- 10% of the voters hold each position
- Voters will vote for the candidate who holds the closest views
- If the candidates hold the same view, they’ll split the vote

Is this a game?
Political Spectrum

Assume:

- There is a spectrum of 10 points on a certain political issue
- There are two candidates
- 10% of the voters hold each position
- Voters will vote for the candidate who holds the closest views
- If the candidates hold the same view, they’ll split the vote

Is this a game?

- Have players (the candidates)
Political Spectrum

Assume:

- There is a spectrum of 10 points on a certain political issue
- There are two candidates
- 10\% of the voters hold each position
- Voters will vote for the candidate who holds the closest views
- If the candidates hold the same view, they’ll split the vote

Is this a game?

- Have players (the candidates)
- Have strategies (1 – 10)
Assume:
- There is a spectrum of 10 points on a certain political issue
- There are two candidates
- 10% of the voters hold each position
- Voters will vote for the candidate who holds the closest views
- If the candidates hold the same view, they’ll split the vote

Is this a game?
- Have players (the candidates)
- Have strategies (1 – 10)
- Need payoffs: choose the % of the vote that they earn
Political Spectrum

- Are there any dominated strategies?

- 1 is weakly dominated by 2
- 10 is weakly dominated by 9
- 3 does not dominate 2 but after we remove 1 it does (assuming common knowledge)
- If we iterate this, the candidates end up in the central positions

This is The Median Voter Theorem

"Majority rule voting will select the median preference"
Political Spectrum

- Are there any dominated strategies?
  - 1 is weakly dominated by 2
     - 10 is weakly dominated by 9
     - 3 does not dominate 2 but after we remove 1 it does (assuming common knowledge)
- If we iterate this, the candidates end up in the central positions
  - This is the Median Voter Theorem
    - "Majority rule voting will select the median preference"
Political Spectrum

- Are there any dominated strategies?
  - 1 is weakly dominated by 2
  - 10 is weakly dominated by 9
- Anything else?
  - 3 does not dominate 2 but after we remove 1 it does (assuming common knowledge)
  - If we iterate this, the candidates end up in the central positions
    - This is the Median Voter Theorem
      - "Majority rule voting will select the median preference"
Political Spectrum

- Are there any dominated strategies?
  - 1 is weakly dominated by 2
  - 10 is weakly dominated by 9
- Anything else?
  - 3 does not dominate 2
    but after we remove 1 it does (assuming common knowledge)
Political Spectrum

- Are there any dominated strategies?
  - 1 is weakly dominated by 2
  - 10 is weakly dominated by 9
- Anything else?
  - 3 does not dominate 2
    but after we remove 1 it does (assuming common knowledge)
- If we iterate this, the candidates end up in the central positions

"Majority rule voting will select the median preference"
Political Spectrum

- Are there any dominated strategies?
  - 1 is weakly dominated by 2
  - 10 is weakly dominated by 9

- Anything else?
  - 3 does not dominate 2
    - but after we remove 1 it does (assuming common knowledge)

- If we iterate this, the candidates end up in the central positions

- This is **The Median Voter Theorem**
  - “Majority rule voting will select the median preference”