A CLASSIFICATION OF GENUS 0 MODULAR CURVES WITH A RATIONAL POINT

Let $E$ be a non-CM elliptic curve defined over $\mathbb{Q}$. Fix an algebraic closure $\overline{\mathbb{Q}}$ of $\mathbb{Q}$. We get a Galois representation

$$\rho_E : \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \text{GL}_2(\hat{\mathbb{Z}})$$

associated to $E$ by choosing a compatible bases for the $N$-torsion subgroups of $E(\overline{\mathbb{Q}})$. Associated to an open subgroup $G$ of $\text{GL}_2(\hat{\mathbb{Z}})$ satisfying $-I \in G$ and $\det(G) = \hat{\mathbb{Z}}^\times$, we have the modular curve $(X_G, \pi_G)$ over $\mathbb{Q}$ which loosely parametrises elliptic curves $E$ such that image of $\rho_E$ is conjugate to a subgroup of $G'$. In this talk I will discuss my work on classification of all such genus 0 modular curves that have a rational point. This classification is given in finitely many families. Moreover, each such modular curve can be explicitly computed.

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