Math 602 Assignment 5, Fall 2020

Part A. From Gallier–Shatz:

- problem 58
- problem 69, 70 (basics of sheaves, presheaves and abelian categories)
- problem 76
- problem 78

Part B.

1. Let A be the polynomial ring $\mathbb{Z}[x, y]$. Compute explicitly the A-modules $\mathrm{Tor}_i^A(A/(xA + yA), A/(xA + yA))$ and $\mathrm{Ext}^i_A(A/(xA + yA), A/(xA + yA))$, for all $i \geq 0$.

2. Let $R = \mathbb{Q}[\mathbb{Z}/3\mathbb{Z}]$ be the complex group ring of the cyclic group $\mathbb{Z}/3\mathbb{Z}$.

   (a) Classify all irreducible R-modules up to isomorphisms. (Recall that an R-module M is irreducible if $M \neq (0)$ and the only R-submodules of M are M and (0).)

   (b) For each pair if irreducible R-modules M, N in the list of irreducible R-modules you found in (a), compute $\mathrm{Tor}_i^R(M, N)$ and $\mathrm{Ext}^i_R(M, N)$.